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Numerical modelling of electromagnetic induction in the Earth magnetic polarization

Numerické modelování pole elektromagnetické indukce při magnetické polarizaci

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Abstract: The basis for interpretation of magnetotelluric observations is the knowledge of the distribution of electromagnetic field for two- and three-dimensional geoelectrical models. For some models analytical solutions are available while for others there are only numerical methods of solution of induction problem and, among them, particularly differential methods. For two-dimensional (2-D) models the cases of electric and magnetic polarization must be considered separately. In the paper algorithm for computation of electromagnetic field, induced with the plane wave, is given for two-dimensional model and magnetic polarization. Geometrical and physical parameters of the model are arbitrary. The results of the computations are given as the curves of magnetotelluric profiling and soundings. The computing program is written in Turbo C language for IBM PC-AT microcomputers.

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Introduction

The effectivity of magnetotelluric methods is much increased with the knowledge of distribution of electromagnetic field for two- and three-dimensional geoelectrical models. The essence of the problem is the solution of the Helmholtz equation for the particular polarization of the plane wave as the field source. Analytical solutions are possible for the models having simple geometrical forms (Rikitake 1960). Such models include cylindrical or spherical body buried in a layered medium (Kaufmann-Keller 1981). The solution of the problem needs the relation of field presentation in rectangular coordinate system and in spherical or cylindrical systems. The effect is obtained by representing the plane wave as a sum of spherical or cylindrical waves. Analytical solutions have limited range of application but, on the other hand, they allow to establish explicitly the relations between field characteristics and physical and geometrical parameters of the studied model. Accuracy of the analytical solutions may be determined in relatively simple way.

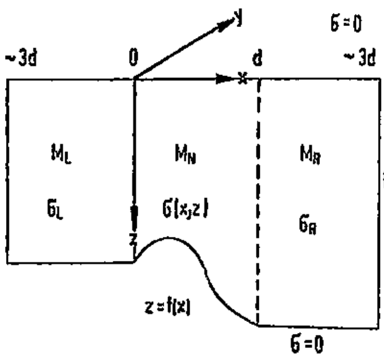
For the models with complex geometrical forms and arbitrary electrical conductivity, numerical methods are effective. Of these, differential methods are widely applied. There are several solutions in these methods. The most general case is when the node of approximating net lays on the contact of four different conductivities $\sigma_1, \sigma_2, \sigma_3, \sigma_4$ or corresponding wave numbers k_1, k_2, k_3, k_4. In the below . In the below given algorithm for electromagnetic field the method of the Varga approximation (1962) of the Helmholtz equation is used.

Differential methods may be used for arbitrary distribution of conductivity in geoelectrical model. The difficulties are related with the solution of large system of equations generated in the whole region. For electric and magnetic polarization of primary field,completely different boundary conditions are formulated for upper and lower border of the region.

In the paper the case of magnetic polarization is discussed.

Algorithm of electromagnetic field computation

Let us consider two-dimensional geoelectrical model in which the axis of homogeneity agrees with the axis of the Cartaesian x, y, z coordinate system (+ z axis is directed vertically down). The primary field is the plane electromagnetic wave harmonically dependent on time t, i.e. exp. $(-i\omega t)$, where ω is frequency of field changes. Electric conductivity of the medium is the function of position, $\sigma = \sigma(x, z)$. Assuming that the field induced in the medium does not change along y-axis, electric polarization has the following components



1. Two- dimensional geoelectrical model. x, y, z – rectangular coordinate system; d – width of inhomogeneity; M_N, M_L, M_R – inhomogeneous, left and right regions; σ_L, σ_R – electric conductivity; h_L, h_R – depth of high resistivity basement.

$$E^E(0, E_y, 0)$$

$$H^E(H_x, 0, H_z) \qquad (1)$$

and for magnetic polarization

$$E^H(E_x, 0, E_z)$$

$$H^H(0, H_y, 0) \qquad (2)$$

On the Earth's surface $E_z = 0$.

It may be easily proved that field components directed along the homogeneity axis satisfy the following differential equation

$$(\Delta 2 + k^2)\, E_y = 0 \qquad (3)$$

for electric polarization, and

$$\frac{\partial}{\partial x}\left(\frac{1}{\sigma}\frac{\partial H_y}{\partial x}\right)+\frac{\partial}{\partial z}\left(\frac{1}{\sigma}\frac{\partial H_y}{\partial z}\right)+i\,\omega\,\mu\,H_y=0 \tag{4}$$

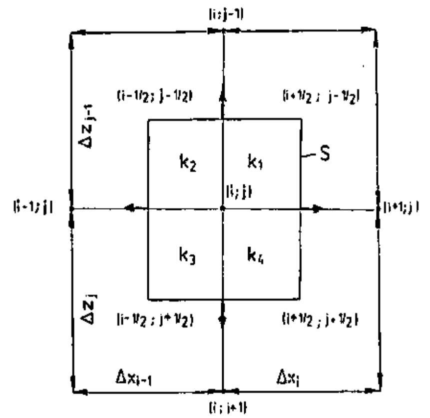
for magnetic polarization.

In the above given equations $\Delta\, z$ stands for two-dimensional Laplace operator and $K^2 = i\,\omega\,\mu\,\sigma$ is a wave number of the medium. It may be easily found that for σ = const, equation (4) becomes Helmholtz equation.

We divide the studied medium into three regions. The central region, M_N, has width d and is the region of inhomogeneity. The boundaries of the left, h_L, and right, h_R, regions are shifted from the boundaries of inhomogeneous region for such the distance that the distortion due to inhomogeneity can be neglected and one-dimensional model, depending only on z-coordinate, can be assumed on the boundaries. In the upper and lower parts of the region we assume the medium for that $\sigma = 0$ (Fig. 1). Onto the constructed region the net is superimposed whose nodes lie on the contact of four different conductivities $\sigma_1, \sigma_2, \sigma_3, \sigma_4$ (Fig. 2). We replace Helmholtz equation in the nodes of the net by differential approximation. It is much simplified if Helmholtz equation is integrated over the area S

$$\int_S \Delta\, 2 A\, dS = -\int_S k^2 A\, dS \tag{6}$$

where A is an arbitrary amplitude of the field. The integral on left-hand side is transformed using Green's equation after Piskorek (1980)



2. Diagram of the net approximating the Helmholtz equation. i,j – coordinates of the net node; $\Delta\, x$, Δz – net size, K_1, K_2, K_3, K_4.-wave number; S – area of the net.

$$\int_S \Delta\, 2 A\, dS = \int_L \frac{\partial A}{\partial n}\, dL \tag{7}$$

L is the boundary of the region S, and n is its external normal.

The continuity of E_x and E_z components for magnetic polarization is the continuity of

$$\sigma^{-1}\,\frac{\partial H_y}{\partial z} \quad \text{and} \quad \sigma^{-1}\,\frac{\partial H_y}{\partial x},$$

respectively. Thus, equation (6) must have the form

$$\int_L k^{-2} \frac{\partial H_y}{\partial n}\, dL = -\int_S H_y\, dS \qquad (8)$$

Substituting central differences for normal derivates and putting them into equation (8) and assuming that magnetic field inside the region S is constant and has the value of $H_y(i,j)$, we have

$$H_y(i,j) = C_{ij}^H \left[E_{ij}^H \,.\, H_y(i+1,\,j) \;+\; N_{ij}^H\;(i,j-1) \;+\; W_{ij}^H \,.\, H_y(i-1,j) \;+\; S_{ij}^H \,.\, H_y(i,j+1) \right] \qquad (9)$$

where

$$E_{ij}^H = \frac{k_1^{-2}\;\Delta z_{j-1} + k_4^{-2}\;\Delta z_j}{2\,\Delta x_i}\;;\; N_{ij}^H = \frac{k_2^{-2}\;\Delta x_{i-1} + k_1^{-2}\;\Delta x_i}{2\,\Delta z_{j-1}}$$

$$W_{ij}^H = \frac{k_2^{-2}\;\Delta z_{j-1} + k_3^{-2}\;\Delta z_j}{2\,\Delta x_{i-1}}\;;\; S_{ij}^H = \frac{k_3^{-2}\;\Delta x_{i-1} + k_4^{-2}\;\Delta x_i}{2\,\Delta z_{j-1}}$$

$$C_{ij}^H = \left[E_{ij}^H + N_{ij}^H \;+\; W_{ij}^H + S_{ij}^H \;-\; \frac{1}{4}\,(\Delta x_i + \Delta x_{i-1})\,(\Delta z_j + \Delta z_{j-1}) \right]^{-1} \qquad (10)$$

The boundary conditions are relatively simply formulated for magnetic polarization.

On the upper boundary of the region, for $z = 0$, and having $E_z = 0$, from equation

$$(\nabla \times H)_z = \sigma E_z = 0$$

we get

$$H_y\;(x,0) = \text{const.} \qquad (11)$$

On the lower boundary, with insulating basement, we assume

$$H_y\;(x, z = f(x)) = 0 \qquad (12)$$

On the left and right boundaries of the region we assume, as it was already mentioned, one-dimensional model. The component $H_y(z)$ must satisfy Helmholtz equation as well as conditions (11) and (12). Thus, we take

$$H_y(z) = C\, sh\; ik\;(h - z) \qquad (13)$$

where C is an arbitrary constant.

In each internal node of the region we get the linear equation on account of unknown values of magnetic field. In the whole region, i x j linear equations are generated that can be written in matrix form

$$A \,.\, x = B \qquad (14)$$

where A is an adjugate, five-diagonal matrix.

Matrix B is expressed in terms of coefficients E_{ij}^H, N_{ij}^H, W_{ij}^H, S_{ij}^H, and C_{ij}^H and field

values on the boundary of the region. Column vector x is the vector of searched field values $H_y(ij)$.
Electric field is calculated from equation

$$E_x = \frac{-1}{\sigma}\frac{\partial H_y}{\partial z} \tag{15}$$

The z-coordinate derivate of magnetic field intensity is computed numerically. Then, for electric and magnetic field, we compute apparent resistivity and phase shift according to relation

$$\rho_T = \frac{1}{\omega\mu}\left|\frac{E_x}{H_y}\right|^2 \quad ; \quad \varphi_T = |Arg|\ \frac{E_x}{H_y} \tag{16}$$

Theoretically, the most reasonable method of solution of the equation system (14) is Gauss-Seidel method, but in practice, it proved to be too slow. Thus, system of equations (14) is solved using the overrelaxation method which gives the expected results in the problem. Before iteration process, the system (14) must be conditioned in relation to diagonal element A_{ii}. After the operation, iteration loop has the form

$$\Delta H_y^n(i,j) = -\alpha\left[H_y^n(i,j) + E_{ij}H_y^n(i+1,j) + N_{ij}H_y^n(i,j-1) + W_{ij}H_y^n(i-1,j) + S_{ij}H_y^n(i,j+1)\right]$$

$$H_y^{n+1}(i,j) = H_y^n(i,j) + \Delta H_y^n(i,j) \tag{17}$$

where n is the number of iteration, and α is the parameter of overrelaxation.The error of successive iteration is defined as

$$\varepsilon^n = \sum^{N_x-2}\sum^{N_z-2} |\Delta H_y^n(i,j)| \tag{18}$$

The iteration process is stopped when $\varepsilon^n < 10^{-6}$. The proper choice of the overrelaxation parameter α is essential for optimization of parameter and speed of computations. The applied method of solution of the linear equations system enables the computations starting from the value$\lambda_1/h_1 = 5$, where $\lambda_1 = (10^7\ \rho_1\ T)^{1/2}$ is the wavelength in the medium with resistivity ρ_1, and T is the period of field variations. Below this boundary wave-length, the system of equations is improperly conditioned and the process becomes divergent. In the range of short waves the initial value of overrelaxation parameter must be carefully chosen. For longer waves, the value of α practically does not depend on the wave length and the previously computed value of overrelaxation parameter may be taken as the starting value in computation of the field value shorter wave.

The program is written in Turbo C language for microcomputer IBM PC. The actual version of the program allows using the nets N_x x $N_z \leq 7200$ (N_x and N_z – numbers of nodes in a net). The speed of computation is comparable to the speed of programs in FORTRAN.

Results of computations

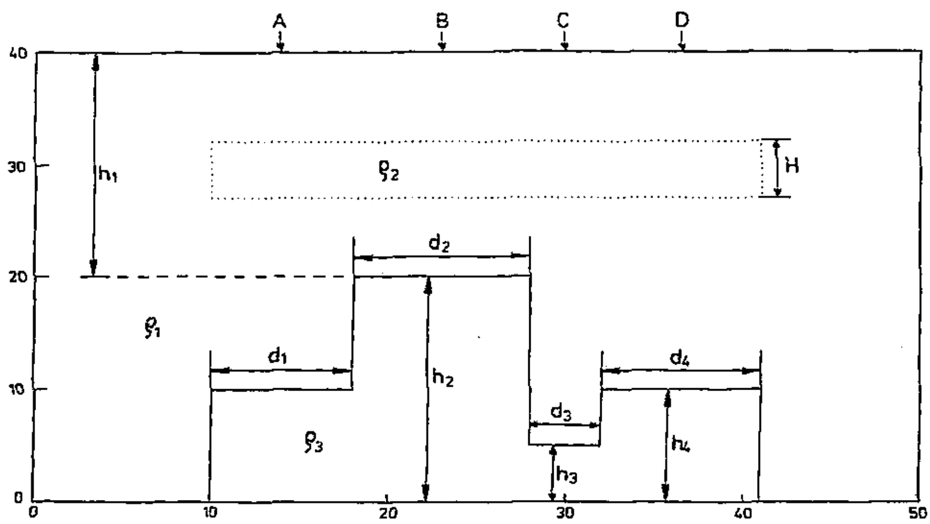
The geoelectric cross-section, taken into computations, is shown in Fig. 3. Morphology of high-resistivity basement with resistivity ρ_3 is created by horst of width d_2 and trough of width d_3. Between the Earth's surface and the basement there is a horizontal layer with thickness H and resistivity ρ_2. Resistivity of the environment is ρ_1. The variable parameters of the model are: d_3/d_2 and ρ_2/ρ_1. The remaining parameters are constant and have the values: $h_1 = h_2 = 7.94$ unit; $h_3 = H = 1/2.\ h_4$; $h_4 = 1/2.h_1$.

Such geoelectric model enables to analyse the reproducibility of morphology of high resistivity basement and the problem of screening of electromagnetic field with the intermediate layer.

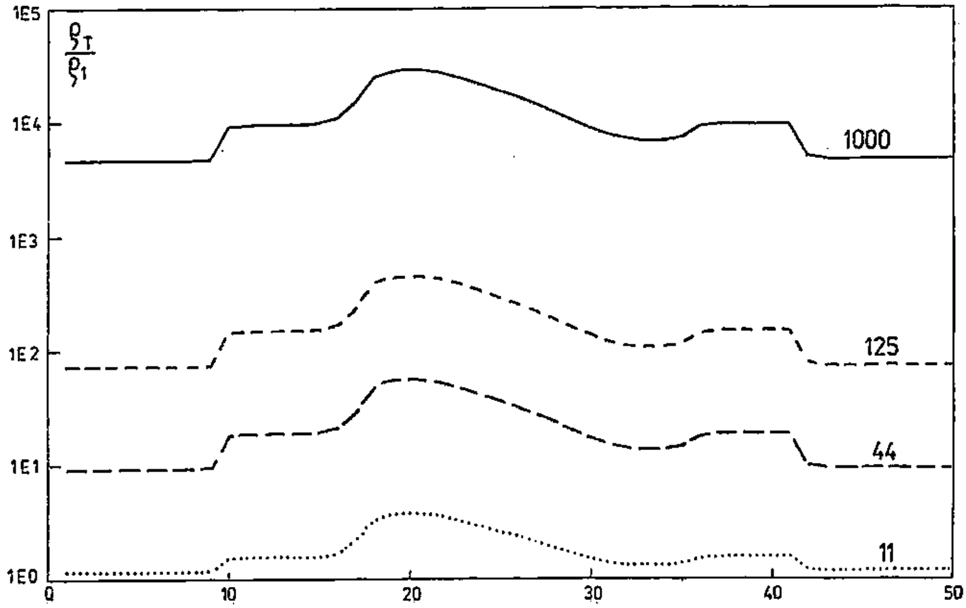
Electromagnetic response of the imposed geoelectric model is presented as the plots of apparent resistivity versus position, for given wave length (profiling) and as the curves of magnetotelluric soundings in points A, B, C, D (see Fig. 3).

If $\rho_2 = \rho_1$ i. e. there is no intermediate layer in geoelectric cross-section, then electromagnetic field changes are related mainly to the changes of morphology of high resistivity basement. The rate of its reproducibility for various wave lengths is shown in Fig. 4. In the range of low frequencies, impedance is inversely proportional to the longitudinal conductance of the overburden and, hence, all elevations of high resistivity basement are reflected in the field image.

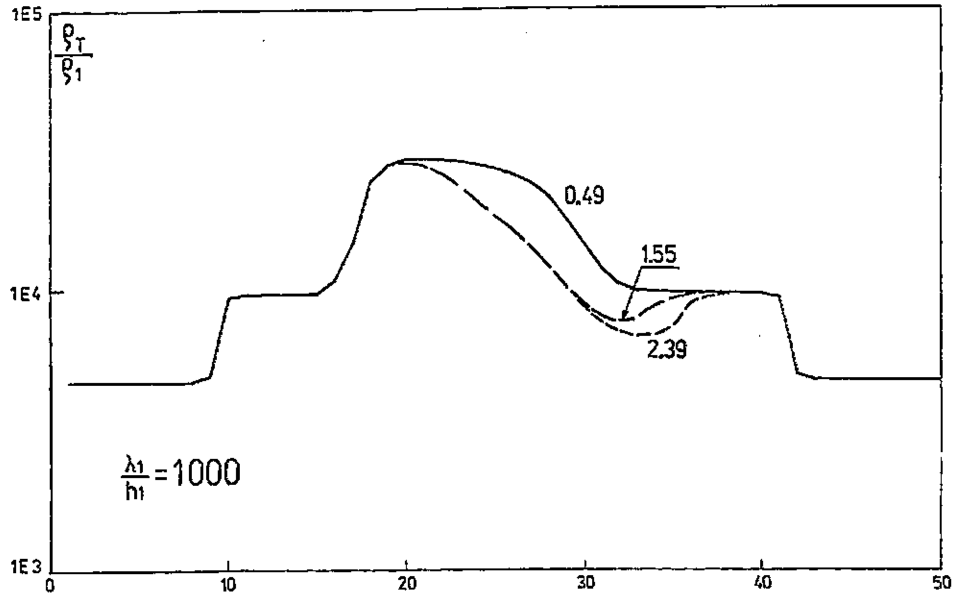
The rate of reproducibility of basement depressions depends on the ratio of their widths to the length of electromagnetic wave and to the width of the surrounding elevations. In the case of narrow depressions, the structure is flown round by electromagnetic field and the depressions cannot be registered. This is shown in Fig. 5.



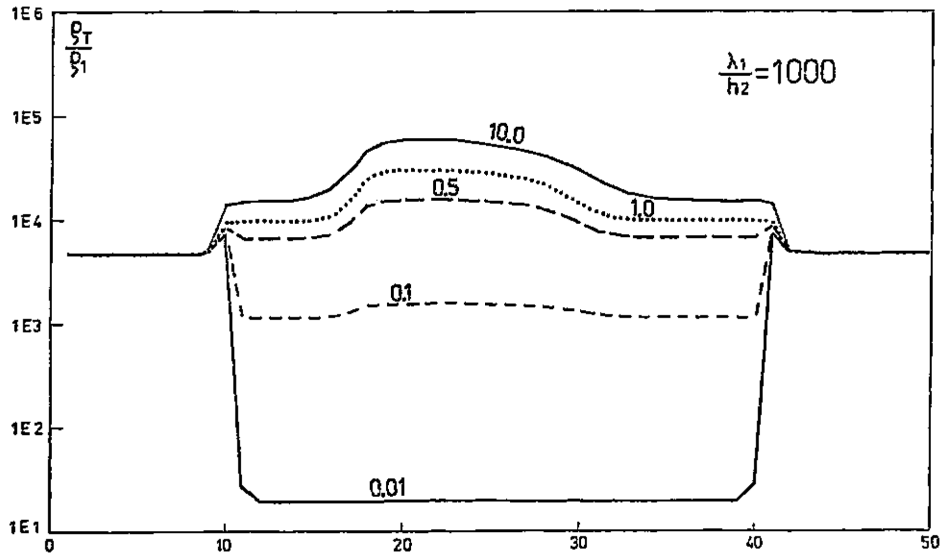
3. Geoelectrical cross-section used in computations. 10, 20, 30 – number of net nodes; A, B,C, D – sounding sites.



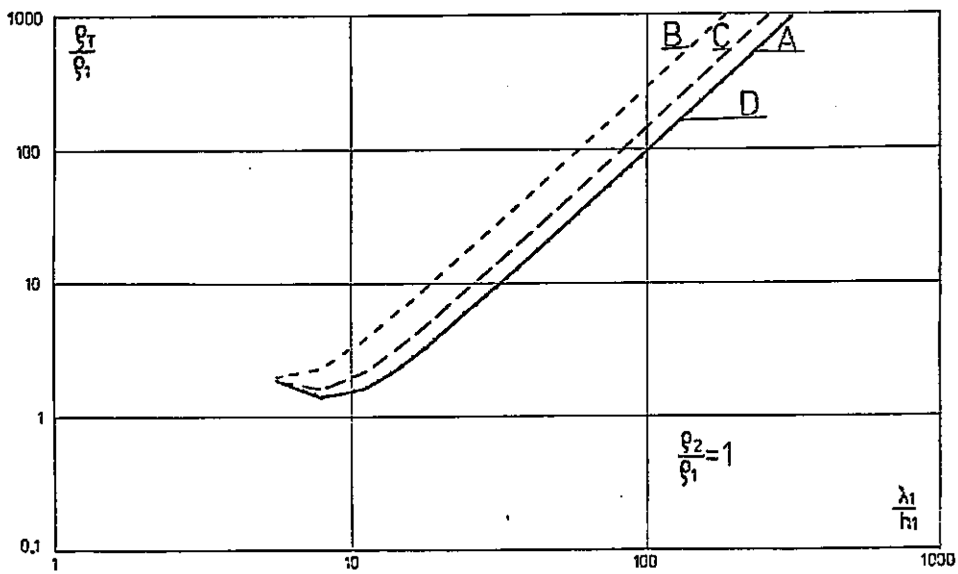
4. Magnetotelluric profiling curves. d_3/d_2 – 2.39, $\rho_2 = \rho_1$, parameter of the curves –relative electromagnetic wave length.



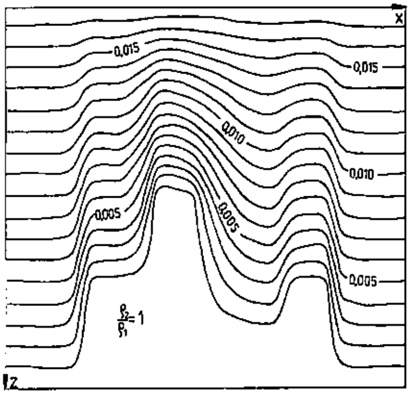
5. Magnetotelluric profiling curves. $\lambda_1/h_1 = 1000$; parameter of the curves – d_3/d_2; $\rho_2/\rho_1 = 1$.



6. Magnetotelluric profiling curves. λ_1/h_1 =1000; parameter of the curves – ρ_2/ρ_1 .



7. Amplitude magnetotelluric sounding curves. $\rho_2/\rho_1 = 1$; A, B, C, D – sounding sites.



8. The changes of magnetic field intensity. $\rho_2 = \rho_1$; 0.005 – value of isoline of magnetic field intensity; x, z – coordinates of rectangular system.

Low resistivity intermediate layer acts as electromagnetic screen which damps the effects from the basement. The rate of screening depends, of course,on the ratio of layer resistivity to environment resistivity, what is shown in Fig. 6. The presence of high resistivity layer in geoelectric cross-section (particularly for low frequencies) does not lead to significant distortion of the image of high resistivity basement (see Fig. 6).

The sounding curves, computed for various points of the profile, generally behave similarly as in the case of one-dimensional model. The shape of the curves depends mainly on resistivity changes in vertical plane in the sounding point. Right, low resistivity asymptotes of amplitude sounding curves, reflect, first of all, the value of longitudinal conductance of the overburden in the sounding point. The rule does not apply to the sounding sites lying over narrow depressions of high resistivity basement, because of the above mentioned effect of electromagnetic field flowing around the structure (Fig. 7).

The changes of magnetic field amplitude in vertical plane x z are presented in Fig. 8, for geoelectric model without intermediate layer. On lower and upper boundary of the region, magnetic field satisfies the imposed boundary conditions. It is seen from the field image that in the regions where high resistivity horizon lies horizontally, magnetic field for low frequencies changes with depth approximately linearly.

The effect may be possibly used to develop the direct method of computing the depth of high resistivity basement.

K tisku doporučil J.Gruntorád
Přeložil P. Borzemski

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Numerické modelování pole elektromagnetické indukce při magnetické polarizaci

(Resumé anglického textu)

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Základem pro interpretaci magnetotelurických měření je znalost rozdělení elektromagnetického pole pro dvou- a třírozměrné geoelektrické modely. Pro některé modely je známo analytické řešení, zatímco pro jiné známe pouze diferenciální postupy řešení problému indukce. V případě dvourozměrných modelů musí být elektrická a magnetická polarizace uvažována odděleně. V práci je podrobněji popsán algoritmus pro výpočet elektromagnetického pole, vyvolaného rovinnou vlnou pro případ dvourozměrného modelu a magnetické polarizace. Rozměry a fyzikální parametry modelu jsou volitelné. Výsledkem výpočtu jsou teoretické křivky pro magnetotelurické profilování a sondování. Program byl napsán v jazyce Turbo C pro osobní počítač IBM AT.

Vysvětlivky k obrázkům

1. Dvourozměrný geoelektrický model. x, y, z – pravoúhlý souřadnicový systém; d – šířka nehomogenity; M_N, M_L, M_R - nehomogenní, levostranná a pravostranná oblast; σ_L, σ_R – elektrická vodivost; h_L, h_R - hloubka vysokoodporového podloží.
2. Diagram aproximace Helmholtzovy rovnice. i, j – souřadnice síťových uzlů; Δx, Δz – velikost sítě; k_1, k_2, k_3, k_4 – vlnové číslo; S – plocha sítě.
3. Geoelektrický řez, použitý při výpočtu. 10, 20, 30 – body uzlů sítě;A, B, C, D – umístění sond.
4. Křivky magnetotelurického profilování. d_3/d_2 – 2,39; $\rho_2 = \rho_1$; parametr křivek – relativní délka elektromagnetické vlny.
5. Křivky magnetotelurického profilování. λ_1/h_1 = 1000; parametr křivek – d_3/d_2; $\rho_2/\rho_1 = 1$.
6. Křivky magnetotelurického profilování. λ_1/h_1 = 1000; parametr křivek – ρ_2/ρ_1.
7. Křivky amplitud magnetotelurického sondování. $\rho_2/\rho_1 = 1$; A, B, C, D – umístění sond.
8. Změny intenzity magnetického pole. $\rho_2 = \rho_1$; 0,005 – hodnota izolinie intenzity magnetického pole; x, z – souřadnice pravoúhlého systému.