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## Mathematical-statistical study of chemical composition of the teschenite association rocks

### Matematicko-statistická studie chemického složení hornin těšínitové asociace

Pavel Machek<sup>1</sup> - Dalibor Matýsek<sup>1</sup>

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**Abstract:** The presented work continues in efforts to establish a classification of teschenite association rocks based on their chemical composition using mathematical-statistical methods. To processing of a data set of 122 chemical analyses taken from technical literatura, methods of multidimensional analysis have been applied. The standardized data set was then processed by cluster analysis with the Ward-Wishart clustering strategy, by analysis of principal components and fuzzy clustering. By the given seven categories representing the fundamental petrochemical rock types of the studied set have been defined. In this way, the fundamentals for classification of studied rocks in the triangle system of three finite elements were established.

The mathematical-statistical apparatus used in processing the chemical analyses of teschenite association rocks has proved a sufficient sensitivity and differentiating abilities in classifying the analytic data into categories, while obtaining other interesting facts.

There exist two principal trends of changes in chemism of the teschenite association rocks - differentiation and assimilation.

The results obtained by multidimensional analysis of chemical composition supplement a petrographic model of teschenite association rock formation.

<sup>1</sup>*Vysoká škola báňská, katedra geologie a mineralogie, tř. 17. listopadu, 708 33 Ostrava-Poruba*

Since the time of the first descriptions of the teschenite association igneous rocks in the first half of the 19th century a number of classifications of these rocks have come into existence. Due to complex mineral and chemical composition of rocks, great variability in structure and texture, and also due to high degree of secondary effects on rocks (such as carbonization, zeolitization, etc.) the teschenite association rock classifications published are difficult to compare and review. A number of new rock terms and their synonyms appeared used only in occurrence areas of individual rocks. Some of the terms have been defined inaccurately or insufficiently. Moreover, the situation is aggravated by mixing mineralogical and structural classification criteria (Smulikowski, 1929). A review of classification approaches according to Šmíd (1978) is given in Table 1.

This study continues in efforts to establish a classification of teschenite association rocks based on their chemical composition using mathematical-statistical methods (Matýsek, 1989). The approach is justified by the authors by the fact that the rocks studied contain a great proportion of microscopically indistinguishable paste (up to 40 vol. %) and that structural features have no decisive

importance for classification. Within the individual rock bodies the structure and texture features vary so much that an unequivocal systematic classification of rock is impossible. This results from specific genetic conditions of such rocks (subaqueous effusions, intrusions into incoherent sediments, existence of volcanoclastic varieties - comp. Matýsek, 1984; Šmíd, 1978).

The aim of this study is to formulate a basic concept of the nature of chemical variability, geochemical development and unified classification and division of the teschenite association rocks.

#### Characteristics of analytic data and methodology of their processing

For mathematical-statistical processing, results of 122 chemical analyses in total have been selected from technical literature - see Table 2 (Pacák, 1926, analyses No. 62-65; Jurková, 1971, analyses No. 95-102; Šmíd, 1978, analyses No. 66-94; Mahmoud, 1973, analyses No. 113-122; Mandour, 1982, analyses No. 1-53; Kudělásková, 1982, analyses No. 103-112). From the

Table 1  
Summary of concepts of teschenite association rock classification (B. Šmíd, 1978)

		B. Šmíd (1975)	K. Smulikowski (1929)
Group porphyric picrite - diabase	feld-spathic - free	porphyric picrite	prolific olivine ankaratrite
		analcimic limburgite ± prolific olivine analcimic augite	augitic ankaratrite (augite)
	feld-spathic	diabasic picrite ± amphibolic	camptonite (?)
		(olivinic) diabase (mandelstone) - (olivinic) dolerite, ophitic to poikilophitic texture	olivinic diabase - olivinic dolerite
Group monchiquite - picrite	feld-spathic - free	olivinic teschenite	camptonite
		picrite	picrite
		teschenitic picrite, amphibolic ouachitite, analcimic scyelite	apatitic ouachitite
	feld-spathic	olivinic monchiquite - monchiquitic picrite	olivinic monchiquite
		(melanocratic) monchiquite - (melanocratic) fourchite, amphibolic, biotite, amphibolic-biotite facies	(melanocratic) monchiquite (melanocratic) fourchite
Group teschenite	feld-spathic	teschenitic monchiquite - teschenitic fourchite biotite to amphibolic-biotite facies	monchiquite
		TESCHENITE subhedral poikilophitic texture, pyroxenic and amphibolic facies, pyroxenic and amphibolic teschenite	theralitic, essexitic, monzonitic teschenite
		analcime-rich facies, lugarite	lugarite
		mafic minerals-rich facies, teschenite, pyroxenite	alkalic pyroxenite bekinkinite
		femic minerals-rich facies "analcimic syenite"	analcimic syenite
		femic minerals-rich facies, rich in Na, K feldspars and nepheline, "nephelinic syenite"	nephelinic syenite

originally published data analyses with CO<sub>2</sub> contents beyond 8 % have been excluded. To suppress the carbonation effect the following calculations have been made in several alternatives. The frequently applied CaO content correction by subtracting a value corresponding to CO<sub>2</sub> content determined has proved as inadequate. Part of carbonates is most likely composed of dolomite-ankerite because corrected CaO contents show negative values in some samples. Another experiment with CaO contents correction to a constant CO<sub>2</sub> content equal to an average of all samples proved more reliable results, but because of probable analytic errors in CO<sub>2</sub> determination in five samples of the set, even this correction alternative could not be applied in the following procedure. A final decision has been taken to eliminate the CaO and CO<sub>2</sub> contents from calculations after a previous conversion of analyses to a constant sum of 100 % in all samples. It is true that by this approach a significant feature is missed but on the other hand the calculation results are not so much affected by secondary processes.

Also the H<sub>2</sub>O<sup>+</sup> and MnO contents have been excluded as they were not determined in all samples and, besides, they cause complications in the result interpretation. The H<sub>2</sub>O<sup>+</sup> and MnO contents are not decisive in the rock classification. The final modification made in input

analytic data was a conversion of the Fe content to a total Fe<sub>2</sub>O<sub>3</sub> content. For processing of such modified data set some of the multidimensional analysis methods have been used.

To suppress the influence of absolute values of individual element contents, the input set standardized prior to further calculations, i.e. in two alternatives. Relatively good results have been obtained by standardization by means of the standard deviation.

$$x_{ij}^S = \frac{x_{ij} - \bar{x}_j}{s_j}$$

where

$x_{ij}$  . . . . . original content value of the  $j$ -th element in the  $i$ -th sample,

$x_{ij}^S$  . . . . . standardized value of this content,

$\bar{x}_j$  . . . . . average of content value of the  $j$ -th element in all samples,

$s_j$  . . . . . the standard deviation of the  $j$ -th element.

In the second alternative the standardization by means of maximum and minimum values of contents of each oxide has been used, i.e.

$$x_{ij}^{MM} = \frac{x_{ij} - x_{jmin}}{x_{jmax} - x_{jmin}}$$

O. Pacák (1926)	J. Klvaňa (1897)	Rohrbach (1885)	G. Tschernak (1865)	L. Hohenegger (1861)
picrite	porphyritic picrite	rocks similar to basalts	picrite	teschenite
teschenitic picrite ± (mandelstone)			picrite (mandelstone)	
olivinic teschenite	(olivinic) diabase - dolerite	diabase (teschenite)	altered picrite altered teschenite	
peridotitic (?) olivinite	picrite	picrite	picrite	
teschenitic peridotite				
picrite				
teschenitic picrite				
analcimic teschenite				
olivinic teschenite				
analcimic teschenite				
analcimic teschenite	teschenite with mafic matrix	teschenite	teschenite	
nephelinic teschenite				
analcimic teschenite	teschenite with fenic matrix	teschenite	teschenite	
teschenitic pyroxenite				
analcimic teschenite				
nephelinic teschenite				

where

$x_{ij}^{MM}$  ... standardized value of the  $j$ -th element content in the  $i$ -th sample,

$x_{jmin}$  ... minimum content of the  $j$ -th element,

$x_{jmax}$  ... maximum content of the  $j$ -th element in the set.

The results were comparable with the first alternative however, a more pronounced "focusing" of the resultant structure and more contrasting definition of rock categories have been achieved. By using this standardization method the newly obtained values have almost an equal "weight" for classification, even if correlations between individual elements are slightly distorted and the interpretation of results may not be so convincing as in the case of the standardization by the standard deviation. (In the analytic data set standardized by standard deviation the average of all oxides equals to zero and the standard deviation equals to one. In the standardization by maximum and minimum values the converted content values of all oxides vary from zero to one). Without standardization the calculation results were determined almost exclusively by the element which has the greatest standard deviation (absolute variability), i.e. in the case of chemical rock analysis by the  $\text{SiO}_2$  value.

Nevertheless, the justification of various standardization methods of output data and applicability of the standardization itself remains to be resolved.

The standardized output analytic data have been further processed by the "classic" cluster analysis with Ward-Wishart clustering strategy, by principal component analysis and fuzzy clustering. The mathematical principles of individual methods and the ways they are applied are widely published including computer programmes (see, for instance, Aitchinson, 1983; Bezdek, 1981; Bezdek et al., 1984; Duran - Odell, 1974; Le Maitre, 1968; Lukasová - Šarmanová, 1985).

The "classic" cluster analysis enable us to find out groups of mutually most similar samples (clusters) in the input set, while the similarity criterion may be for example the Euclidean distance calculated in  $n$ -dimensional space determined by the system of coordinate axes along which original and standardized content values of individual oxides are plotted. By means of the so called fuzzy clustering the probability can be determined by which the input set samples can be classified into individual clusters, and thus the position of transient samples and "outsiders" can be defined. The objective of the principal component analysis is to find out for each object in the  $n$ -dimensional space new coordinates (called principal components)

Table 2  
Input matrix - chemical analyses of teschenite rock association

	SiO2	TiO2	Al2O3	Fe2O3	FeO	MnO	MgO	CaO	Na2O	K2O	F2O5	CO2	H2O+
1 TESCHENITE	40.00	2.80	13.00	6.86	5.30	0.20	4.45	12.77	2.75	3.00	0.75	3.76	4.09
2 TESCHENITE	41.40	2.70	15.40	4.00	6.47	0.18	4.80	11.42	2.75	3.00	0.80	3.90	2.70
3 TESCHENITE	40.50	3.80	12.55	6.55	8.34	0.28	3.52	9.65	2.00	2.50	1.75	5.04	3.06
4 TESCHENITE	40.90	2.60	17.40	3.60	6.47	0.20	2.22	11.42	1.80	2.76	0.80	5.80	3.70
5 TESCHENITE	39.00	3.80	15.64	0.80	5.03	0.20	3.85	16.50	3.20	1.80	1.71	4.80	3.40
6 TESCHENITE	42.90	3.00	10.06	5.11	5.60	0.22	4.80	6.72	4.40	1.42	1.80	0.95	5.30
7 TESCHENITE	37.40	4.70	19.00	6.54	7.04	0.18	3.65	10.75	2.60	0.60	0.85	3.80	2.57
8 TESCHENITE	38.00	3.10	14.00	8.00	5.75	0.35	3.50	11.65	3.50	2.20	1.95	4.20	2.85
9 TESCHENITE	42.00	3.50	19.25	8.15	3.60	0.17	2.00	6.30	4.00	3.52	1.05	1.95	4.60
10 TESCHENITE	34.20	3.50	10.00	5.91	7.46	0.22	4.80	11.20	2.62	1.00	0.75	6.60	2.55
11 TESCHENITE C	44.20	3.15	16.20	2.72	7.48	0.23	3.50	8.51	3.50	2.30	0.95	4.14	2.86
12 TESCHENITE C	39.00	3.70	12.62	7.66	6.15	0.28	3.50	12.10	2.90	2.80	1.57	3.90	4.10
13 TESCHENITE C	37.30	3.95	13.70	4.47	11.07	0.30	2.90	11.54	2.80	0.95	1.50	7.40	1.60
14 TESCHENITE C	41.10	2.90	19.00	5.60	6.47	0.19	4.15	8.05	5.55	1.42	1.20	0.00	4.10
15 TESCHENITE C	40.90	2.70	18.50	5.30	5.45	0.21	4.80	9.41	4.65	2.22	2.25	0.00	3.60
16 TESCHENITE C	42.20	2.90	16.00	8.35	5.03	0.17	4.12	10.75	4.00	1.80	1.05	0.00	3.50
17 TESCHENITE C	43.30	2.90	17.10	7.50	4.31	0.26	3.20	8.05	5.35	1.92	1.20	0.00	4.05
18 TESCHENITE C	41.30	3.00	12.05	6.94	8.32	0.22	5.20	11.04	4.12	1.72	1.75	1.12	3.32
19 TESCHENITE N	41.64	2.16	18.88	3.90	6.86	0.19	2.54	10.28	4.62	3.04	0.92	0.62	4.28
20 TESCHENITE O	39.50	2.30	14.60	6.55	4.31	0.10	7.24	14.30	1.32	2.10	0.71	4.40	3.27
21 TESCHENITE O	43.40	2.05	15.10	6.85	4.17	0.11	8.00	7.62	2.42	0.55	0.32	6.20	3.00
22 TESCHENITE X	38.40	3.30	13.40	5.25	5.90	0.19	5.11	16.35	1.80	2.33	0.65	4.56	2.54
23 TESCHENITE X	41.20	2.60	13.60	5.20	6.75	0.12	7.64	12.20	2.90	0.75	0.65	3.50	2.85
24 TESCHENITE X	40.10	3.10	10.20	7.00	4.17	0.17	4.20	12.40	3.26	1.85	0.65	1.75	2.75
25 TESCHENITE B	37.40	3.75	10.05	6.40	9.34	0.23	6.00	11.00	1.40	1.80	3.40	6.00	2.85
26 TESCHENITE AX	41.80	2.65	13.00	6.40	4.46	0.19	5.44	15.00	2.40	1.70	0.52	1.80	4.30
27 TESCHENITE	43.10	2.75	12.60	10.38	3.73	0.19	2.60	11.65	4.20	0.50	0.50	6.20	2.40
28 TESCHENITE X	38.00	3.40	15.60	2.75	6.90	0.19	6.05	14.55	2.15	2.33	0.63	4.40	2.60
29 TESCHENITE X	38.20	3.40	11.60	5.10	5.90	0.20	5.44	18.14	2.20	1.50	0.50	5.04	2.46
30 TESCHENITE X	40.10	2.65	10.60	5.30	5.03	0.18	4.80	16.14	1.95	2.33	0.52	6.40	1.70
31 NONCHIQUTE	37.10	3.30	11.60	8.94	6.47	0.30	6.40	11.20	2.55	2.00	2.20	4.96	2.94
32 NONCHIQUTE	42.00	2.10	17.90	6.70	4.31	0.17	6.40	12.60	2.55	1.22	0.55	0.30	2.30
33 NONCHIQUTE	38.70	3.50	15.60	7.70	5.60	0.15	6.72	12.10	2.40	1.42	0.60	2.40	2.90
34 NONCHIQUTE	38.00	3.50	13.10	7.20	4.77	0.22	7.65	15.20	2.05	0.65	0.75	2.04	4.56
35 FOURCHITE	33.70	3.00	10.90	9.27	6.75	0.35	5.45	14.33	2.90	1.11	3.13	4.24	3.56
36 FOURCHITE	32.80	3.85	11.15	11.34	6.61	0.31	6.70	13.45	2.30	1.80	2.85	3.80	2.57
37 FOURCHITE	34.70	3.90	10.07	6.40	8.04	0.37	5.77	14.00	2.10	1.70	3.13	6.00	3.40
38 FOURCHITE	35.50	3.95	10.05	9.10	6.32	0.32	3.80	14.11	5.20	1.90	2.37	5.92	1.10
39 FOURCHITE	34.30	3.70	11.40	5.80	12.51	0.18	4.15	11.65	1.30	0.55	2.60	7.44	4.11
40 FOURCHITE	38.60	3.00	12.20	9.60	5.75	0.21	7.00	10.30	3.00	2.65	1.70	0.00	4.75
41 FOURCHITE AB	31.50	3.80	15.30	3.33	11.35	0.14	4.86	12.10	2.30	1.25	2.75	8.72	1.90
42 FOURCHITE AB	36.00	3.10	16.20	4.95	6.47	0.14	8.30	14.33	1.20	1.70	0.77	3.90	2.02
43 QUACHITITE	38.30	4.40	8.27	6.22	3.80	0.36	7.00	16.90	1.80	1.95	4.27	5.51	1.29
44 QUACHITITE	35.10	3.20	12.80	9.55	4.31	0.28	10.22	13.90	0.65	1.11	1.15	5.20	2.20
45 QUACHITITE	31.30	3.92	10.56	10.33	7.60	0.21	7.83	13.47	2.30	1.90	5.01	2.35	2.57
46 PICRITE T	40.10	2.40	8.22	5.74	8.23	0.15	11.30	13.21	2.00	1.50	0.87	2.83	3.20
47 PICRITE	39.40	1.94	7.60	4.90	7.32	0.18	19.02	10.50	1.43	1.62	0.70	0.85	5.04
48 PICRITE	39.70	2.35	8.45	4.40	8.27	0.15	19.06	9.98	1.52	0.75	0.66	0.85	3.56
49 PICRITE	37.60	2.10	13.00	6.62	5.35	0.14	13.11	13.45	0.57	0.39	0.65	3.00	3.10
50 PICRITE	37.10	2.15	11.60	6.05	5.03	0.18	17.20	12.20	0.75	0.55	0.65	4.20	2.22



Table 2 - continuation

51	PICRITE	40.74	2.52	13.10	3.38	8.26	0.19	7.87	8.96	3.68	0.62	0.46	4.77	5.22
52	PICRITE	37.12	3.02	10.02	4.82	5.12	0.19	10.85	14.92	1.32	0.90	0.68	3.02	7.65
53	PICRITE	37.60	2.30	12.50	6.55	3.31	0.18	8.50	13.85	1.66	1.20	0.70	6.05	5.65
54	PICRITE	39.78	2.39	8.41	4.47	8.12	0.15	19.02	9.92	1.66	0.74	0.48	0.00	3.40
55	PICRITE	39.38	1.73	7.64	4.97	7.23	0.17	22.05	8.62	0.43	1.12	0.44	0.00	4.93
56	PICRITE T	40.20	2.48	8.04	4.23	9.66	0.15	11.28	13.23	2.00	1.50	0.81	2.79	2.45
57	PICRITE T	31.20	4.01	10.65	10.23	7.66	0.24	7.30	13.74	2.22	2.11	5.04	2.83	1.82
58	TESCHENITE O	43.30	2.10	10.71	2.88	8.65	0.13	12.63	11.90	2.17	1.21	0.59	0.18	0.59
59	TESCHENITE A	41.42	3.14	15.07	6.40	7.93	0.20	4.83	10.16	4.00	1.98	1.57	0.00	2.73
60	TESCHENITE X	39.52	2.75	9.36	3.74	7.39	0.17	8.64	16.94	1.49	1.63	1.36	3.07	3.01
61	TESCHENITE N	42.46	2.24	19.18	3.83	6.85	0.19	2.63	10.23	4.30	3.13	0.78	0.00	3.35
62	DIABASE	39.73	2.15	12.46	3.73	7.19	0.18	8.26	14.72	2.39	1.83	1.00	3.00	3.28
63	DIABASE	43.55	2.63	13.37	4.69	5.83	0.15	6.95	13.47	1.07	3.17	0.52	1.16	2.51
64	TESCHENITE X	49.00	0.39	14.67	6.40	7.93	0.14	6.69	9.43	3.56	1.90	0.39	0.49	2.31
65	TESCHENITE	47.20	2.22	18.92	3.83	5.60	0.16	3.05	7.56	4.45	2.80	0.69	0.00	3.06
66	MONCHIQUTE	40.68	3.57	11.23	8.87	5.17	0.22	3.12	10.32	2.22	0.68	2.18	5.08	4.92
67	FOURCHITE B	40.24	3.59	10.54	8.95	4.02	0.27	3.06	11.86	1.72	0.82	2.30	6.19	4.46
68	FOURCHITE B	36.68	3.57	10.30	6.18	10.13	0.22	5.58	10.37	1.56	0.37	2.38	5.40	5.48
69	FOURCHITE B	35.90	3.71	10.72	6.02	10.78	0.23	5.41	10.00	1.50	0.36	2.02	5.28	6.05
70	FOURCHITE B	34.84	4.09	10.05	5.48	10.71	0.29	4.43	11.30	1.34	0.75	2.60	6.19	5.55
71	QUACHITITE	34.86	4.00	10.62	6.55	9.95	0.25	4.57	11.34	1.38	0.75	2.50	5.82	5.85
72	QUACHITITE	31.97	3.95	10.31	3.20	13.72	0.29	4.79	12.51	1.40	0.47	2.87	7.65	5.06
73	QUACHITITE	35.52	3.54	9.30	7.95	6.65	0.27	3.16	15.11	1.58	0.77	2.20	9.28	4.24
74	FOURCHITE B	39.10	3.96	11.28	4.52	9.70	0.27	4.06	10.25	2.23	0.65	2.42	6.30	4.63
75	MONCHIQUTE	37.25	3.76	10.96	2.80	13.04	0.17	5.56	9.15	2.44	0.42	2.71	4.81	5.70
76	TESCHENITE	33.38	3.36	8.49	10.47	5.89	0.24	6.85	13.81	1.38	2.10	2.34	5.62	4.32
77	FOURCHITE B	29.60	3.70	8.19	12.41	8.19	0.02	7.55	13.94	2.12	1.50	4.95	3.15	2.35
78	PICRITE	28.49	3.00	8.00	12.84	8.58	0.02	0.14	14.22	1.82	1.65	5.10	3.08	2.40
79	PICRITE	28.99	3.93	8.19	12.40	8.48	0.02	0.39	13.60	1.06	1.86	5.35	1.96	2.65
80	FOURCHITE	36.66	0.94	12.39	7.79	7.87	0.22	9.68	7.37	1.56	0.47	2.14	2.64	7.03
81	FOURCHITE B	38.80	0.96	12.76	4.79	9.48	0.22	9.58	7.72	1.59	0.35	2.16	2.30	6.36
82	FOURCHITE B	28.24	3.93	12.61	2.01	10.01	0.20	7.56	12.76	1.47	0.30	2.82	6.96	5.65
83	FOURCHITE	36.81	3.75	12.55	3.56	9.91	0.20	6.56	9.88	3.24	0.96	1.92	5.56	5.17
84	PICRITE	39.95	3.37	11.75	12.08	5.84	0.24	5.65	9.65	2.39	2.72	1.95	0.44	3.84
85	FOURCHITE B	36.51	3.26	11.48	10.30	5.64	0.36	5.90	9.00	2.80	2.32	3.08	1.96	4.79
86	PICRITE	41.06	2.44	13.08	3.04	8.62	0.10	6.78	8.64	3.31	0.51	0.37	6.09	5.05
87	FOURCHITE A	43.79	2.82	14.85	5.83	4.38	0.18	4.04	6.41	3.08	3.32	0.80	3.55	4.79
88	PICRITE	36.17	2.08	3.60	4.62	4.90	0.20	11.58	15.00	1.21	0.60	0.55	3.11	5.98
89	FOURCHITE	40.44	2.98	13.63	3.34	7.63	0.27	4.37	9.48	2.68	2.91	0.98	6.36	4.37
90	TESCHENITE O	37.42	3.46	13.63	7.73	6.21	0.18	5.44	8.74	2.81	2.21	0.57	4.94	5.01
91	FOURCHITE	34.24	4.15	9.94	4.16	12.02	0.20	4.83	12.46	0.56	0.58	1.31	7.41	5.82
92	PICRITE	35.19	4.40	9.12	7.86	7.15	0.22	9.65	11.58	0.56	1.47	0.90	4.54	5.37
93	PICRITE	39.20	2.08	8.63	4.51	7.26	0.23	17.66	10.61	2.58	1.09	0.20	1.32	4.12
94	PICRITE T	40.00	3.45	12.99	3.75	8.84	0.21	5.69	9.36	2.10	2.50	1.90	3.51	4.32
95	FOURCHITE	30.94	3.76	7.82	9.92	8.90	0.28	9.08	13.32	2.38	1.57	5.45	4.95	1.06
96	FOURCHITE	28.74	4.36	7.22	11.43	8.62	0.20	8.00	15.26	1.42	1.18	5.50	7.12	0.00
97	FOURCHITE	32.23	3.61	8.75	9.53	7.46	0.28	7.48	14.71	1.50	1.84	4.20	7.44	0.00
98	TESCHENITE	43.50	2.98	12.01	6.95	4.63	0.22	5.57	12.69	3.14	1.99	1.13	3.17	0.00
99	FOURCHITE	38.37	3.38	10.75	9.73	5.31	0.27	0.30	13.11	2.60	1.17	2.63	3.26	0.00
100	TESCHENITE	36.00	3.96	11.44	7.54	6.77	0.29	8.58	10.52	2.23	2.00	1.93	7.44	0.00

Table 2 - continuation

101	TESCHENITE	33.55	4.83	8.94	7.82	6.25	0.31	7.13	14.15	2.09	1.57	3.31	7.33	0.55
102	OUACHITITE	34.82	3.79	9.55	7.89	8.54	0.31	7.77	13.57	2.23	1.65	3.70	5.19	0.00
103	PICRITE	39.12	1.60	11.99	5.52	5.57	0.13	16.55	14.13	1.40	0.80	0.45	0.50	2.15
104	TESCHENITE X	41.80	2.75	13.07	5.44	6.62	0.18	7.40	15.12	2.35	1.80	0.90	0.40	2.04
105	FOURCHITE	32.50	3.60	13.60	8.15	7.75	0.28	7.65	11.65	2.10	1.22	3.20	6.40	1.70
106	TESCHENITE	41.80	2.55	14.49	4.60	5.57	0.19	5.88	11.93	2.25	2.50	2.15	2.90	3.08
107	TESCHENITE	42.60	2.50	16.58	6.60	5.42	0.18	4.59	10.27	3.50	1.80	2.45	0.40	2.96
108	TESCHENITE X	38.56	2.30	14.90	7.95	6.02	0.18	6.08	13.32	1.90	2.33	0.43	3.20	2.70
109	TESCHENITE X	40.92	2.44	16.35	4.23	6.32	0.20	6.28	10.66	2.35	2.15	1.95	3.00	3.06
110	TESCHENITE	43.10	1.01	16.93	5.47	6.11	0.18	4.71	11.06	2.40	1.00	0.45	2.50	3.16
111	PICRITE	40.20	2.10	8.71	3.30	7.90	0.16	20.73	8.82	2.50	0.80	0.00	0.00	4.80
112	PICRITE	40.32	1.40	8.71	2.10	8.60	0.16	24.80	7.10	1.20	0.40	0.00	0.00	5.14
113	PICRITE	38.90	1.50	12.40	6.30	4.16	0.05	12.55	15.00	1.76	0.66	1.00	1.25	4.46
114	PICRITE	43.40	1.03	9.77	4.49	5.30	0.05	11.45	14.15	0.60	0.93	0.46	0.60	7.23
115	TESCHENITE	40.26	1.23	14.77	6.48	5.24	0.15	6.24	15.10	2.71	2.01	0.29	1.40	3.68
116	TESCHENITE	38.60	2.30	16.90	4.67	5.86	0.02	5.95	13.26	2.31	1.89	1.74	6.05	0.00
117	TESCHENITE	39.54	2.30	14.16	6.32	4.55	0.01	6.13	13.36	3.29	1.77	1.54	1.20	6.38
118	TESCHENITE	39.30	0.92	13.20	0.81	6.18	0.01	11.45	15.46	2.87	3.18	0.37	6.00	0.00
119	MONCHIQUTE	39.03	1.00	13.80	6.69	5.14	0.09	6.92	17.17	2.06	1.69	0.27	2.00	2.00
120	DIABASE	36.90	1.00	9.45	0.10	6.13	0.04	6.59	13.90	1.60	0.90	1.30	4.50	0.73
121	FOURCHITE	38.00	2.49	13.27	8.06	5.53	0.00	6.53	16.67	3.02	0.65	0.62	3.19	0.00
122	MONCHIQUTE	42.80	1.60	10.06	6.21	4.69	0.00	9.85	17.14	2.92	0.45	1.06	3.20	0.00

Teschenite C - analcimic teschenite, teschenite B - biotite teschenite, teschenite N - nephelinic teschenite, teschenite AX - amphibolic-pyroxenic teschenite, teschenite O - olivine teschenite, fourchite B - biotite fourchite, teschenite X - pyroxenic teschenite, fourchite AB - amphib. - biotite fourchite, picrite T - teschenitic picrite, fourchite A - amphibolic fourchite, monchiquite A - amphibolic monchiquite, monchiquite B - biotite monchiquite. Sources of analytic data are given in the text of the paper.

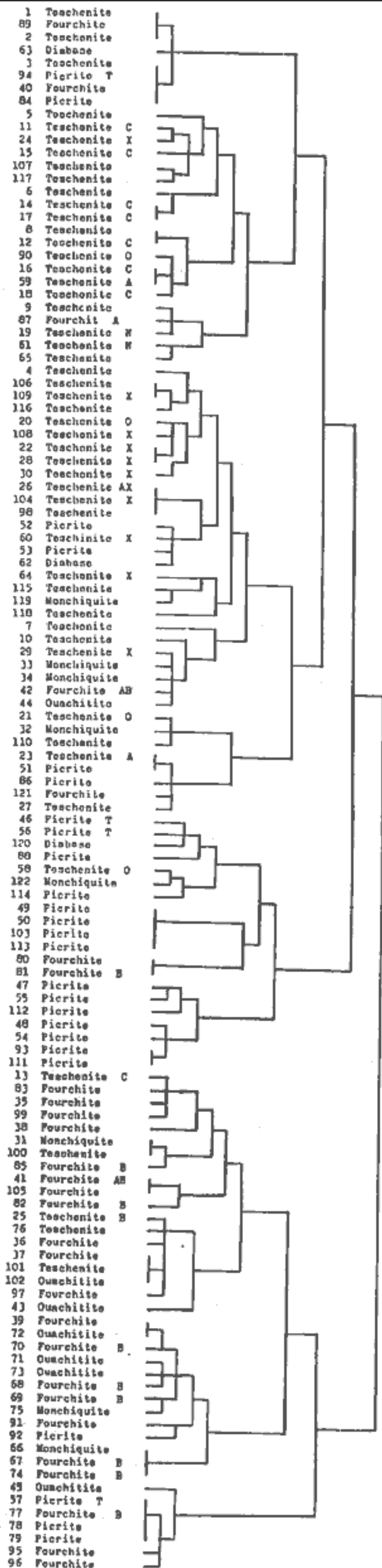
which can be obtained by a linear combination of original, or standardized contents, and are featured by the fact the first few coordinates in sequence characterize substantial part of variability of the analytic data set. By plotting the first versus the second and the first versus the third main component of all samples into diagrams, an effective graphic illustration of the studied data set structure and of the groups defined can be obtained.

The "classic" cluster analysis with results shown in the dendrogram in Table 3 gives a concept of classification of chemical analysis set into seven groups. The composition of these groups was then compared with fuzzy clustering results and with results of principal component analysis and modified accordingly. The final structure of the chemical analysis set together with average contents and variation coefficients in individual groups is shown in Table 4 and also illustrated in Fig. 1. The evaluation of statistical significance of differences between the groups by means of some non-parametric tests (Rock, 1986) proved the justification of such division and structure of groups.

Apart from samples given in Table 4, the set comprises 10 samples which do not belong to any of the defined groups and therefore have the position of "outsiders". These are either samples with extreme contents of some elements, or transient rock types. Also errors in the analytic determination could occur (samples No: 5, 7, 38, 44, 58, 60, 85, 87, 100, 113 - see Table 4).

The actual numbers and vectors of covariant matrix set up for standardized content values of eight elements which have been included into calculations are shown in Table 5. The interpretation of individual principal components is based on these values. The first principal component correlates most strongly with SiO<sub>2</sub>, Al<sub>2</sub>O<sub>3</sub> and alkalis (positively) and with TiO<sub>2</sub>, total Fe content and P<sub>2</sub>O<sub>5</sub> (negatively). Along the first principal component the samples are distributed according to the amount of leucocratic and mafic component and the input file of analyses is subdivided into two subfiles. The first of the two subfiles includes groups 1 to 4, and the second one groups 5, 6 and 7.

The first principal component covers 42,17 % of the input data file variance. In the second principal component the distribution of TiO<sub>2</sub>, Al<sub>2</sub>O<sub>3</sub> and alkalis is reflected the contents of which are increasing in its positive direction (positive correlation) in relation to MgO the contents of which are increasing in the opposite direction (negative correlation). The distribution of samples along the second principal component is most likely the result of olivine and feldspar distribution in the rocks. In this direction continuous transitions between groups 1 to 4 can be observed. The values of the second principal components in groups 6 and 7 differ only slightly. The variability of these component values amounts to 27,64 % of the total variability of the file. The first two principal components together represent substantial part, almost 70 % of total



variability of 8 oxides analyzed in the input file. Further interpretation is limited only to these two most important components. In the direction of the third principal component the variability covers only 11,08 % of the total variance. In this direction differences can be observed especially in the alkali proportion in individual samples within the groups but not between the groups. Therefore, the third principal component has no influence on the division of the input file into groups. The fourth to the eighth principal components together cover the remaining less than 20 % of the total variability and their individual interpretation is no longer unequivocal. It is presumed that they express only variance values of individual elements in samples within the the groups determined and can be influenced by the position of some of the "outsiders". As a result, they do not affect the file structure and further interpretation. Because the interpretation is limited only to the first two principal components, a certain error occurs due to information loss arising from the "outsiders" position; but the "outsiders" can be excluded from fuzzy clustering results. Nevertheless, this error will have, due to the above-mentioned facts, only a minimum influence on the reliability of the file division into groups.

#### Characteristics of groups and distribution of main and trace elements (Table 4)

Group 1 includes 13 samples of various teschenite types (analcimic, nephelinic, and a single sample of pyroxenic teschenite). These samples are characterized by high alkali contents, with Na<sub>2</sub>O prevailing over K<sub>2</sub>O. The SiO<sub>2</sub> contents are the highest in the whole file, while the MgO contents are the lowest.

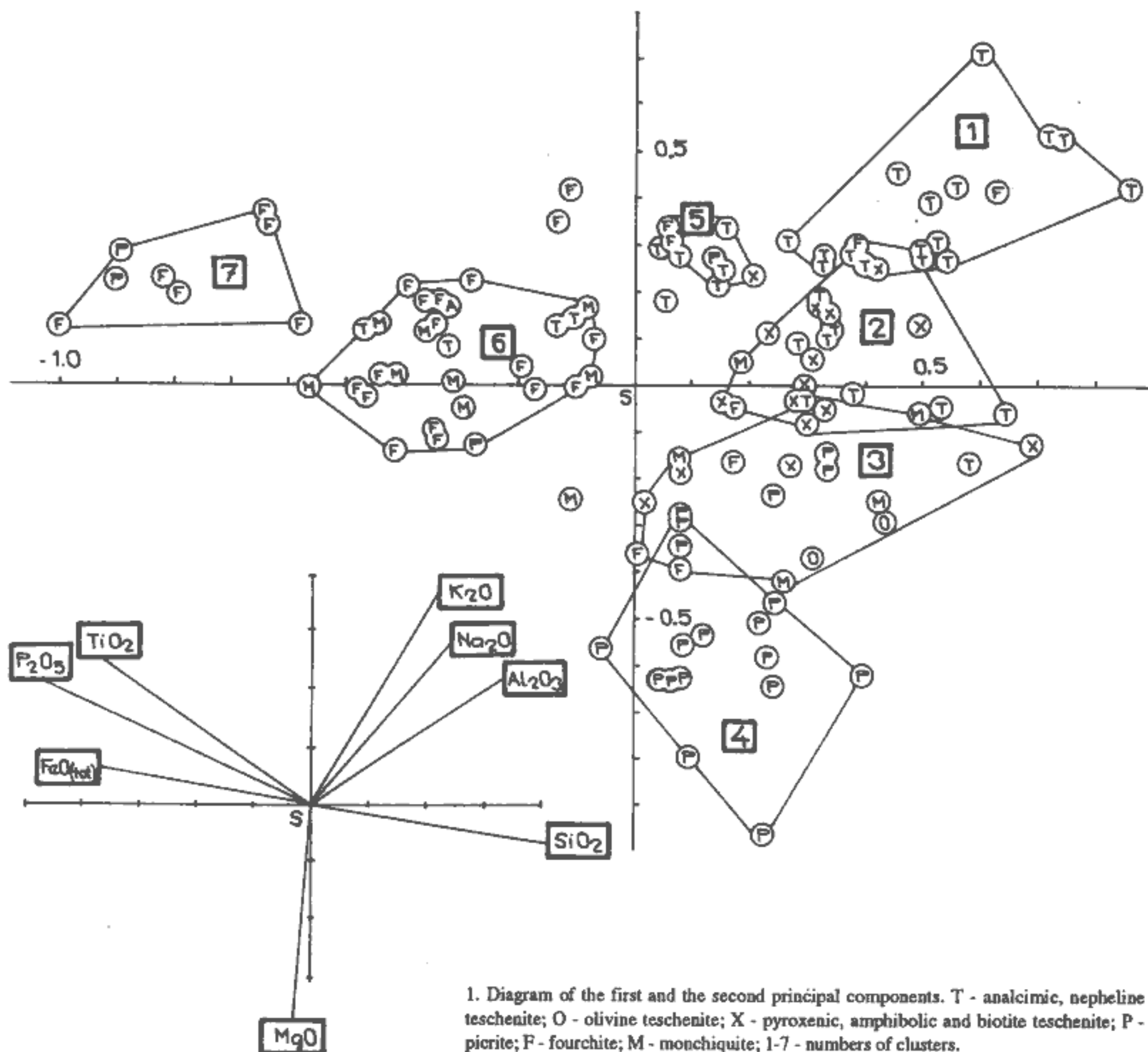
Group 2 comprises 23 samples of various rock types with prevailing teschenites, olivine teschenite and monchiquite are less frequent. Their chemism is featured by higher MgO contents and lower alkali, Al<sub>2</sub>O<sub>3</sub> and SiO<sub>2</sub> contents than in group 1.

Group 3 includes 16 samples represented by monchiquites, diabase picrites, olivine and pyroxenic teschenites and fourchites. From the chemical point of view the MgO contents are rising while the alkali and Al<sub>2</sub>O<sub>3</sub> contents are decreasing. The P<sub>2</sub>O<sub>5</sub> and TiO<sub>2</sub> contents are also slightly decreasing.

Group 4 consists of 15 samples of various picrite and peridotite types. Rocks with higher MgO contents and minimum alkali, TiO<sub>2</sub> and P<sub>2</sub>O<sub>5</sub> contents are represented here.

Group 5 is featured by sharply defined cluster of 9 samples, particularly of various teschenite types. The MgO and alkali contents are similar to group 1 but differ

Table 3  
Dendrogram of teschenite association rocks



in basicity and by sharply increased contents of  $\text{TiO}_2$ ,  $\text{P}_2\text{O}_5$  and total Fe.

Group 6 includes 28 samples of monchiquites, fourchites and ouachitites and 2 teschenite samples. The group differs from groups 1-4 by higher  $\text{TiO}_2$ ,  $\text{P}_2\text{O}_5$  and total Fe contents as well as by high basicity. The alkali contents can be compared with the group 4, the MgO contents with groups 1 and 2.

Group 7 comprises only 8 relatively extreme samples of ouachitites and fourchites. The  $\text{P}_2\text{O}_5$  contents in these rocks are several times higher than in samples of other groups. Also the  $\text{TiO}_2$  and total Fe contents are extremely high. The  $\text{SiO}_2$  contents in these samples are the lowest in the whole file. The alkali contents are in the middle of the whole set. Samples in this group contain a high percentage of ore minerals.

Generally it can be concluded that the MgO contents

are sharply increasing from group 1 to group 4, obviously depending on the negative direction of the second principal component; there is only a slight increase in the contents from group 1 through groups 5 and 6 to group 7. Within the groups the MgO content changes are indistinct with the exception of groups 4 and 6. It is especially in group 4 that a high gradient of MgO contents can be observed.

The alkali contents decrease sharply with the increasing MgO content again along the second principal component, i.e. from group 1 continuously to group 4, while within the group 4 this drop is the most intensive. The differences in the alkali content between groups 6 and 7 are not sharp, within the group 6 a pronounced division of samples depending on alkali contents is evident. The  $\text{Al}_2\text{O}_3$  distribution has an analogic development.

The  $\text{SiO}_2$  contents are considerably changing along the



**Table 4**  
Division of the file into groups and their statistic characteristics

group 1							
sample: 6, 9, 11, 14, 15, 16, 17, 19, 24, 59, 61, 65, 107							
	content (%)						
	SiO <sub>2</sub>	TiO <sub>2</sub>	Al <sub>2</sub> O <sub>3</sub>	Fe <sub>tot</sub>	MgO	Na <sub>2</sub> O	
average	42.563	2.807	17.806	12.070	3.733	4.292	
deviation	1.722	0.397	1.352	1.287	0.932	0.655	
variat. coef. (%)	4.047	14.135	7.592	10.666	24.975	15.252	
	content (%)						
	K <sub>2</sub> O	P <sub>2</sub> O <sub>5</sub>	CaO	CO <sub>2</sub>	MnO	H <sub>2</sub> O	
average	2.251	1.230	9.153	0.816	0.194	3.626	
deviation	0.648	0.525	1.670	1.158	0.026	0.764	
variat. coef. (%)	28.767	42.654	18.295	164.338	13.402	21.070	
	content (ppm)						
	V	Cr	Co	Ni	Ba	K/Rb	Th/U
average	104.857	49.457	21.343	52.142	343.333	259.8	2.417
deviation	51.687	66.150	6.28	22.177	86.15	180.97	1.442
variat. coef. (%)	49.293	133.753	29.424	42.532	25.092	69.646	59.661
group 2							
sample: 1, 2, 4, 20, 22, 26, 28, 29, 30, 33, 42, 62, 63, 89, 98, 104, 106, 108, 109, 115 - 118							
	content (%)						
	SiO <sub>2</sub>	TiO <sub>2</sub>	Al <sub>2</sub> O <sub>3</sub>	Fe <sub>tot</sub>	MgO	Na <sub>2</sub> O	
average	40.164	2.612	14.225	11.575	6.151	2.303	
deviation	1.739	0.611	1.668	1.286	1.727	0.512	
variat. coef. (%)	4.331	23.395	11.726	11.110	28.083	22.218	
	content (%)						
	K <sub>2</sub> O	P <sub>2</sub> O <sub>5</sub>	CaO	CO <sub>2</sub>	MnO	H <sub>2</sub> O	
average	2.256	0.886	13.787	3.668	0.163	2.756	
deviation	0.528	0.493	2.116	1.740	0.064	1.438	
variat. coef. (%)	23.410	55.719	15.348	47.437	39.264	52.177	
	content (ppm)						
	V	Cr	Co	Ni	Ba	K/Rb	Th/U
average	82.73	113.27	32.136	112.55	592.5	367.5	2.55
deviation	92.6	137.70	7.69	84.83	232.7	85.84	0.637
variat. coef. (%)	111.93	121.568	23.93	75.36	39.27	23.36	24.98
group 3							
sample: 21, 23, 27, 32, 34, 51, 53, 64, 80, 81, 86, 110, 119 - 122							
	content (%)						
	SiO <sub>2</sub>	TiO <sub>2</sub>	Al <sub>2</sub> O <sub>3</sub>	Fe <sub>tot</sub>	MgO	Na <sub>2</sub> O	
average	40.934	1.911	13.461	13.154	7.419	2.604	
deviation	2.732	0.820	2.083	1.749	1.880	0.774	
variat. coef. (%)	6.675	42.911	15.472	13.294	25.339	29.712	
	content (%)						
	K <sub>2</sub> O	P <sub>2</sub> O <sub>5</sub>	CaO	CO <sub>2</sub>	MnO	H <sub>2</sub> O	
average	0.842	0.807	11.95	3.503	0.158	3.622	
deviation	0.430	0.599	3.351	1.887	0.059	2.471	
variat. coef. (%)	51.056	74.224	28.042	53.868	37.342	66.731	

Table 4 - continuation

	content (ppm)						
	V	Cr	Co	Ni	Ba	K/Rb	Th/U
average	118.0	395.0	32.0	182.5	940.0		
deviation	92.0	65.0	3.0	97.0	790.0		
variat. coef. (%)	77.966	16.456	9.375	53.151	84.043		
<b>group 4</b>							
sample: 46, 47, 48, 49, 50, 52, 54, 55, 56, 88, 93, 103, 111, 112, 114							
	content (%)						
	SiO <sub>2</sub>	TiO <sub>2</sub>	Al <sub>2</sub> O <sub>3</sub>	Fe <sub>tot</sub>	MgO	Na <sub>2</sub> O	
average	36.642	2.137	9.025	12.460	16.520	1.429	
deviation	1.512	0.542	2.092	1.361	4.299	0.647	
variat. coef. (%)	3.814	25.384	23.185	10.927	26.020	45.291	
	content (%)						
	K <sub>2</sub> O	P <sub>2</sub> O <sub>5</sub>	CaO	CO <sub>2</sub>	MnO	H <sub>2</sub> O	
average	0.925	0.518	11.785	1.543	0.159	4.331	
deviation	0.382	0.261	2.512	1.414	0.038	1.652	
variat. coef. (%)	41.256	50.457	21.315	91.640	23.899	38.144	
	content (ppm)						
	V	Cr	Co	Ni	Ba	K/Rb	Th/U
average	46.5	973.142	65.0	707.571	1.885		1.777
deviation	23.21	470.27	17.07	303.96	115.0		0.184
variat. coef. (%)	49.914	48.325	26.262	42.958	6.101		10.354
<b>group 5</b>							
sample: 3, 8, 10, 12, 18, 40, 84, 90, 94							
	content (%)						
	SiO <sub>2</sub>	TiO <sub>2</sub>	Al <sub>2</sub> O <sub>3</sub>	Fe <sub>tot</sub>	MgO	Na <sub>2</sub> O	
average	39.165	3.509	13.408	15.107	4.962	2.845	
deviation	2.005	0.265	1.788	0.978	1.165	0.623	
variat. coef. (%)	5.120	7.551	13.338	6.474	23.486	21.884	
	content (%)						
	K <sub>2</sub> O	P <sub>2</sub> O <sub>5</sub>	CaO	CO <sub>2</sub>	MnO	H <sub>2</sub> O	
average	2.283	1.555	10.41	3.306	0.243	3.756	
deviation	0.363	0.492	1.079	2.151	0.049	0.813	
variat. coef. (%)	15.878	31.679	10.365	65.064	20.165	21.645	
	content (ppm)						
	V	Cr	Co	Ni	Ba	K/Rb	Th/U
average	31.667	26.667	23.500	56.0	410.0	219.667	2.0
deviation	9.43	7.364	8.860	29.978	92.017	73.893	0.294
variat. coef. (%)	29.778	27.615	37.702	53.532	22.442	33.639	14.700
<b>group 6</b>							
sample: 13, 25, 31, 35, 36, 37, 39, 41, 43, 44, 66-76, 82, 83, 91, 92, 99, 101, 102, 105							
	content (%)						
	SiO <sub>2</sub>	TiO <sub>2</sub>	Al <sub>2</sub> O <sub>3</sub>	Fe <sub>tot</sub>	MgO	Na <sub>2</sub> O	
average	35.776	3.828	11.021	16.341	5.913	1.875	
deviation	2.616	0.298	1.599	1.880	1.878	0.675	
variat. coef. (%)	7.312	7.795	14.505	11.506	31.753	36.011	

Table 4 - continuation

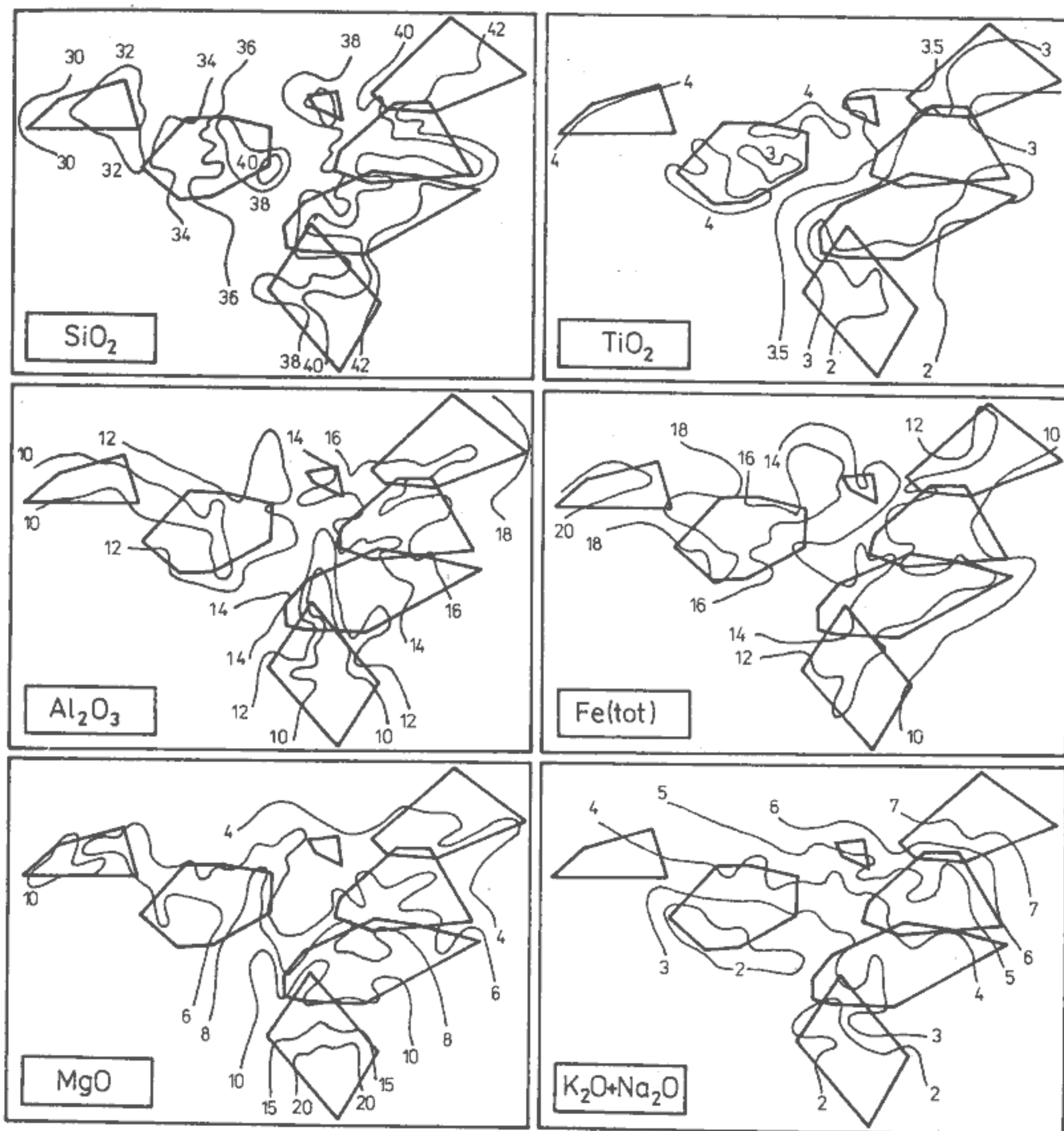
	content (%)						
	K <sub>2</sub> O	P <sub>2</sub> O <sub>5</sub>	CaO	CO <sub>2</sub>	MnO	H <sub>2</sub> O	
average	1.088	2.563	12.267	5.987	0.262	3.681	
deviation	0.545	0.725	1.735	1.350	0.055	1.844	
variat. coef. (%)	50.113	28.305	14.144	22.549	20.992	50.095	
	content (ppm)						
	V	Cr	Co	Ni	Ba	K/Rb	Th/U
average	73.50	15.28	26.60	38.20	330.889	455.0	2.786
deviation	39.69	7.753	5.731	7.43	181.03	185.0	0.741
variat. coef. (%)	54.00	50.739	21.545	19.450	54.71	40.659	26.597
group 7							
sample: 45, 57, 77, 78, 79, 95, 96, 97							
	content (%)						
	SiO <sub>2</sub>	TiO <sub>2</sub>	Al <sub>2</sub> O <sub>3</sub>	Fe <sub>tot</sub>	MgO	Na <sub>2</sub> O	
average	30.570	3.933	8.786	20.417	8.079	1.996	
deviation	1.220	0.214	1.166	1.697	0.545	0.337	
variat. coef. (%)	3.991	5.446	13.276	8.312	6.747	16.903	
	content (%)						
	K <sub>2</sub> O	P <sub>2</sub> O <sub>5</sub>	CaO	CO <sub>2</sub>	MnO	H <sub>2</sub> O	
average	1.742	5.137	14.033	2.544	0.169	1.616	
deviation	0.274	0.392	0.623	2.038	0.117	1.051	
variat. coef. (%)	15.718	7.627	4.439	80.110	69.231	65.037	

Samples not included in the collection: 5, 7, 38, 44, 58, 60, 85, 87, 100, 113

Table 5  
Eigenvalues and eigenvectors of covariation matrix

component No.	1	2	3	4	5	6	7	8
characteristic numbers	14.92	9.78	3.92	2.32	1.70	1.24	0.93	0.57
the share of the whole deviation (%)	42.17	27.64	11.08	6.57	4.80	3.50	2.62	1.61
cumulative (%)	42.17	69.82	80.89	87.46	92.27	95.77	98.39	100.00
characteristic vectors: SiO <sub>2</sub>	0.46	-0.07	-0.06	0.14	-0.40	-0.49	-0.20	0.56
TiO <sub>2</sub>	-0.35	0.35	-0.17	-0.52	-0.53	0.28	-0.04	0.31
Al <sub>2</sub> O <sub>3</sub>	0.36	0.31	-0.34	-0.18	0.62	0.23	0.05	0.42
Fe <sub>tot</sub>	-0.40	0.12	-0.07	0.27	0.06	-0.30	0.75	0.30
MgO	-0.04	-0.49	0.33	0.24	-0.03	0.60	0.05	0.48
Na <sub>2</sub> O	0.25	0.38	-0.32	0.61	-0.34	0.40	0.09	-0.21
K <sub>2</sub> O	0.24	0.53	0.80	-0.05	-0.01	0.01	0.14	0.01
P <sub>2</sub> O <sub>5</sub>	-0.50	0.31	0.08	0.41	0.23	-0.10	-0.60	0.22

Covariation matrix was set up for input data standardized by means of minimum and maximum values.



2. Isolines of the main and some trace element contents in graphs of the first and the second principal components. Group 1-7 are indicated according to fig. 1.

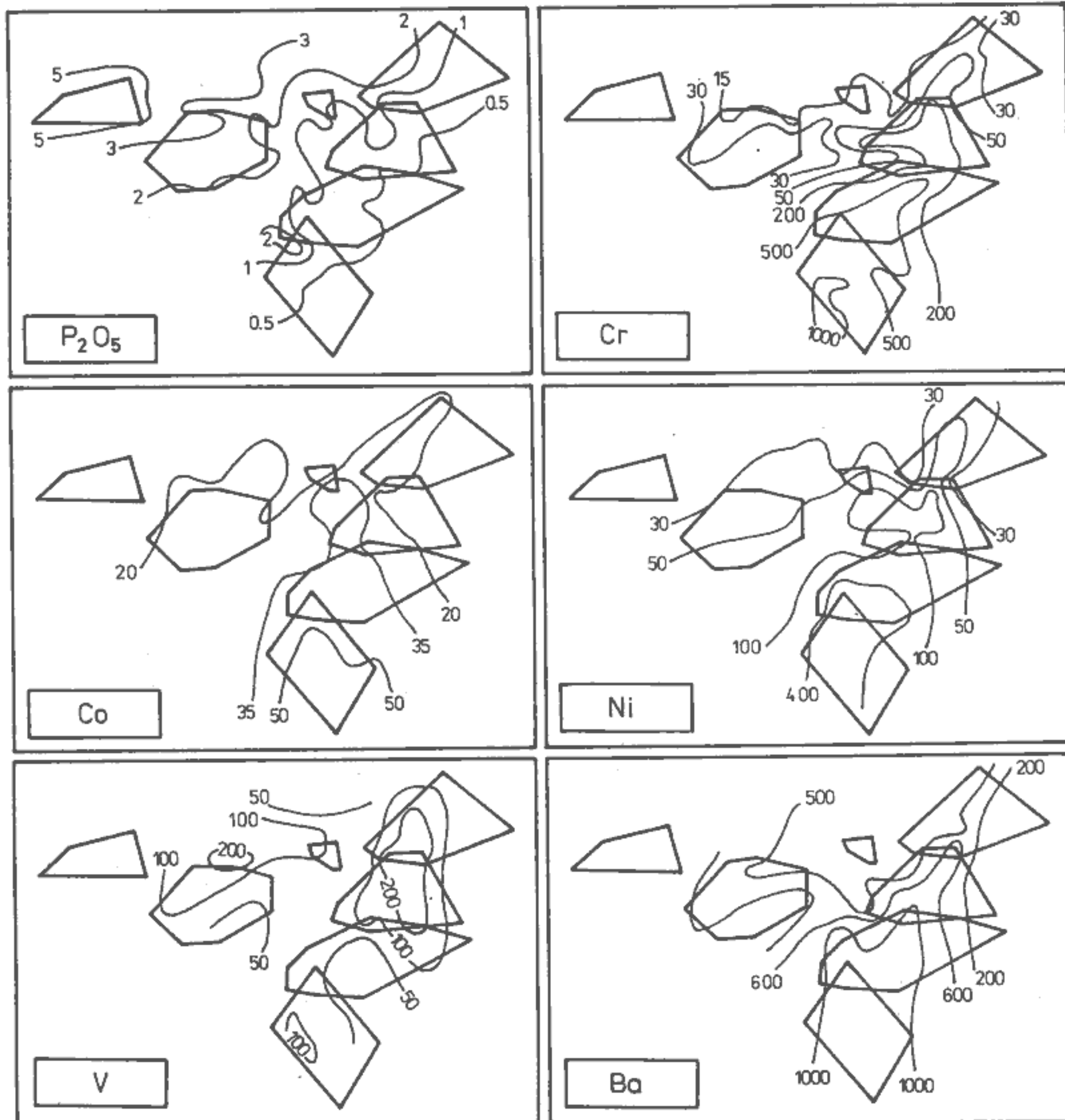
first principal component: they are dropping from groups 1 to 4 through group 6 to group 7. There is no great difference between groups 1-4 but within these groups the samples are divided according to changing contents of  $\text{SiO}_2$  in the direction of the above-mentioned trend.

The variations of the total Fe contents correspond to the type of  $\text{SiO}_2$  changes (with an opposite sign), the changes in the  $\text{TiO}_2$  and  $\text{P}_2\text{O}_5$  contents have analogic development. The maximum gradient has a direction corresponding to

the direction of the first principal component, whereas in the direction of the second principal component only a slight content fluctuation can be observed. Thus there are no substantial differences between groups 1 to 4 with continuous changes from groups 1 to 4 to the group 7 being the most pronounced. Also the total Fe,  $\text{TiO}_2$  and  $\text{P}_2\text{O}_5$  contents are featuring slight changes in the direction of the given trend between samples within the groups 1 to 4.

The variations in values of  $\text{SiO}_2$ ,  $\text{TiO}_2$ ,  $\text{Al}_2\text{O}_3$ , total Fe,

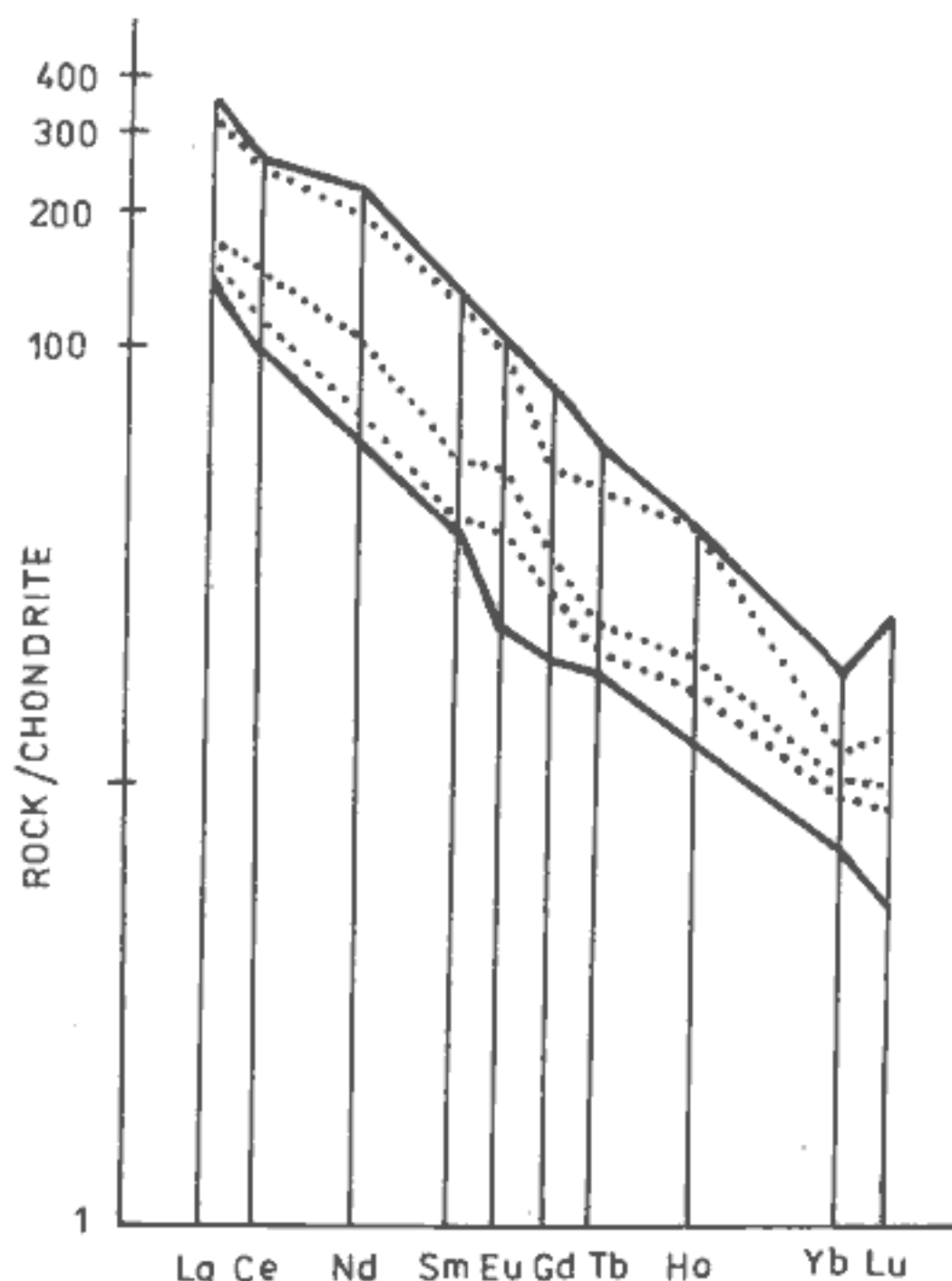




MgO, alkali and other trace element contents within and between the groups are illustrated in Fig. 2.

The structure of chemical analytic data file of teschenite association expressed by the first and the second principal component is represented by the system with triangular polarity (see Fig. 4). One of the extremes are rocks of teschenite composition with maximum alkali and  $Al_2O_3$  and minimum MgO contents. Another extreme are picrites with maximum MgO contents and minimum alkali and

$Al_2O_3$  contents. The third extreme are fourchites and monchiquites with maximum  $P_2O_5$ ,  $TiO_2$  and total Fe contents and minimum  $SiO_2$  contents. Between these extreme rock types there are continuous transitions in relation to the given trends of individual elements, i.e. between groups 1 to 4 and between groups 1, 5, 6, 7. Transitions between groups 4 and 6 are not ruled out but in the input analytic data file such samples are not represented (except for the sample No. 44).



3. Variation range and average distribution curves of TR contents for some of the classified rocks within the teschenite association. Analyses taken from the study by J. Kudělásková (1987). Average TR distribution within the group: 1 - picrites; 2 - pyroxenites; 3 - monchiquites. Normalized according to J.A. Haskin et al. (1986).

The facts stated are confirmed by an analysis of trace element distribution (Cr, Co, Ni, V, Ba, TR). However, the trace elements have not been determined in a sufficient number of samples so as to apply the same method of processing. The evaluation of trace element distribution is based on the analysis of their variance within groups and in the level of the first and the second principal components.

The average trace element content values are given in Table 4. The trends of Cr, Ni and Co contents are similar. They are characterized by maximum contents in group 4 and by a decrease to minimum values in group 1. The trace element contents in group 6 are low, corresponding to contents in group 1. Trends of their changes correspond to MgO changes, they can be correlated with olivine distribution in such rocks. The vanadium contents are maximum in groups 1 and 2 and in group 6. Barium has an almost identical trend with chrome, nickel and cobalt. Maximum values occur in the samples of group 4, the group 1 samples have minimum contents. The rare earth element contents cannot be evaluated statistically because of very small number of analyses. Standardized distribution curves indicate the same characteristics in all

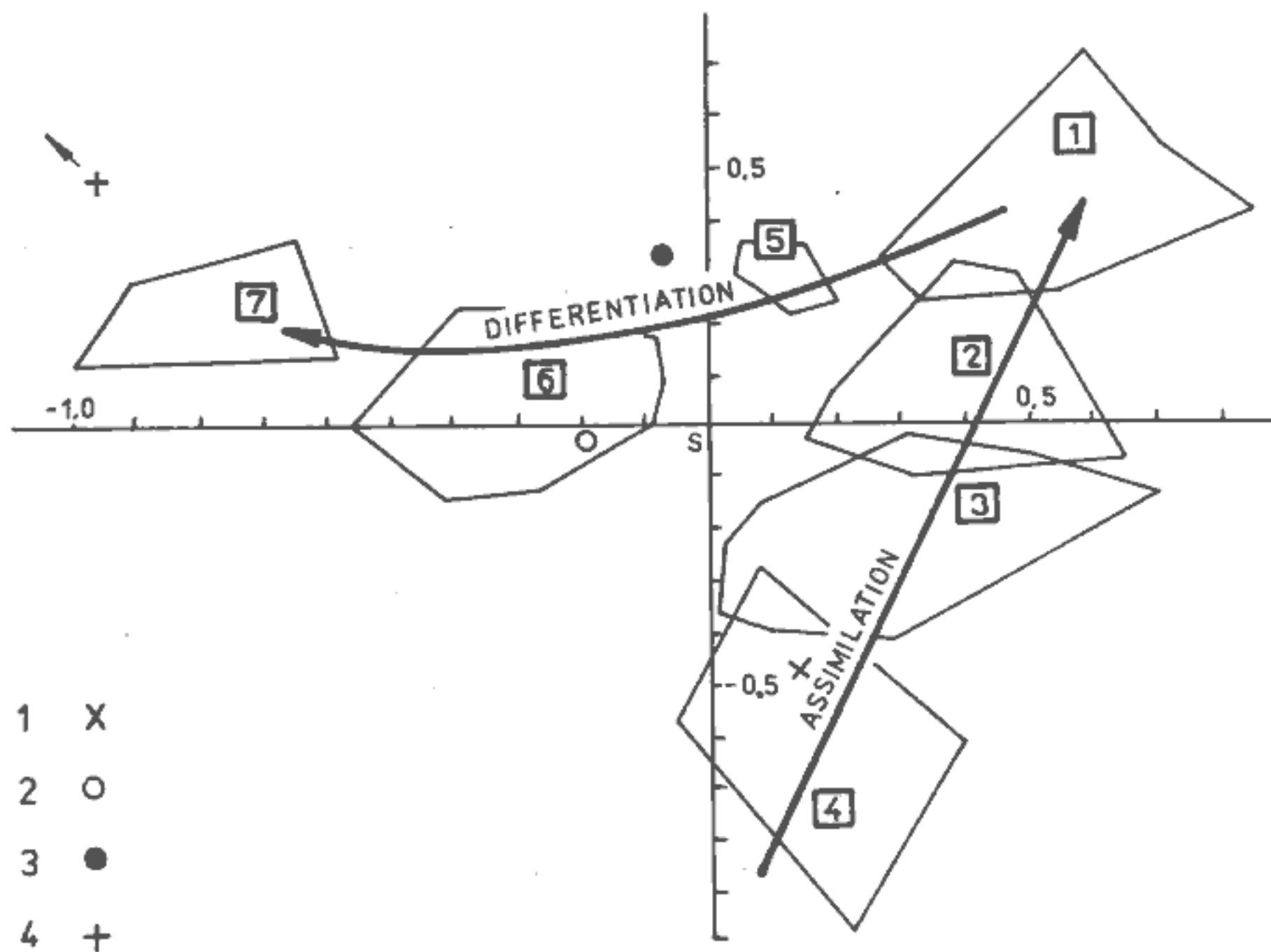
analyses - a continuous, almost linear drop (Fig. 3). No favourable anomalies occur. Distribution curves feature a similar course as those of continental alkali basalts (Gladkich, 1987). The given analyses indicate a rather differentiated material (for instance, La enrichment is hundredfold in relation to chondrites). From the comparison of contents in individual groups it can be seen that the lowest contents occur in group 4 while towards group 1 they slightly increase. The maximum rare earth element contents occur in samples from group 6, which proves their maximum differentiation.

### Interpretation of variability factors

Based on the results obtained by cluster analysis and principal components analysis, important conclusions can be drawn on the variability factors of the chemical analysis file studied indicating the genesis of teschenite association rocks.

The first principal component expresses the basicity changes in these rocks which is affected by increasing or decreasing of the  $TiO_2$ ,  $P_2O_5$  and total Fe contents. Samples included in group 6 and particularly 7 are enriched with ore minerals which is proved by projecting the average amphibole, biotite and pyroxene composition into the level of the first and second principal component (see Fig. 4). The projection of average apatite composition with regard to groups 6 and 7 position and to directions of increasing Fe, Ti and P contents indicates that in the first principal component influences of differentiation processes are reflected (accumulation of ore minerals and apatite).

The second principal component features especially changes in Mg contents in relation to alkali and aluminium (MgO decrease is continuously compensated by increasing the  $Na_2O$ ,  $K_2O$  and  $Al_2O_3$  contents). In this direction there are only slight basicity changes (basicity changes between groups 1 to 4 are much smaller than within these groups). The differences in the composition of groups 1 to 4 can be explained by assimilation processes. By means of elementary concentration equations it can be shown that after subtracting a certain portion of picrite average chemical composition from the average composition groups 1 to 4 a specific portion of residual material always remains with practically unchanged chemism (see Matýsek, 1989a, b). The proportion of residual material is continuously increasing from group 4 to group 1. The average composition of this residual substance is close to chemical composition of rocks referred to as alkali syenites which occur in the area studied very rarely (see Table 6). Only three reliable silicate analyses of these rocks were available which when processed together with multidimensional methods would play the role of extreme outsiders and could distort final results. For this reason they were not included into the



4. Interpretation of variability factors based on the first and the second principal components. 1 - pyroxene; 2 - amphibole; 3 - biotite; 4 - apatite + magmatite.

input analytic data file. However, it should be remembered that in the continuation of group 1, approximately in the direction of rising alkali and  $Al_2O_3$  contents, existence of another separate group can be anticipated at the vertex of triangular structure of analytic data studied.

The first and the second principal component are approximately parallel with the direction of two presumed factors with decisively affect the chemical composition of the rock samples studied, i.e. differentiation and assimilation. The influence of differentiation processes

can be observed along the first principal component, while the assimilation influence is almost perpendicular to this direction. This fact indicates that there are two independent processes. On the one hand the classification of samples into groups 1-4 is probably a result of assimilation, and on the other hand the differentiation processes are functioning as subordinate within these groups. Contrary to these, the differences between the groups 1-4 and 5, 6 and 7 indicate a reverse proportion of the two factors.

Table 6  
Chemical analyses of alkali syenites

sample	author	contents (%)						
		SiO <sub>2</sub>	TiO <sub>2</sub>	Al <sub>2</sub> O <sub>3</sub>	Fe <sub>2</sub> O <sub>3</sub>	FeO		
1.	Mahmoud 1973	46.70	0.78	19.77	3.47	2.70		
2.	Mahmoud 1973	48.50	0.28	18.60	4.61	1.91		
3.	Jurková 1971	50.41	0.55	22.60	1.20	2.74		
sample	contents (%)							
	MnO	MgO	CaO	Na <sub>2</sub> O	K <sub>2</sub> O	P <sub>2</sub> O <sub>5</sub>	CO <sub>2</sub>	H <sub>2</sub> O <sup>+</sup>
1.	0.05	3.01	6.14	5.10	6.20	0.00	2.00	3.64
2.	0.05	1.06	8.90	3.16	6.46	0.02	0.00	6.00
3.	0.11	5.50	6.48	4.23	5.75	0.16	0.00	4.90

## Conclusion

Mathematical-statistical apparatus employed for processing of chemical analyses of teschenite association rocks has proved a sufficient sensitivity and differentiating ability in classification of analytic data into groups. At the same time, further interesting facts have been found out:

1. Basic criteria for classification of these rocks in the triangular system of three finite elements have been successfully defined. The samples representing end-members are not yet available in the accessible analytic data, but their composition can be estimated with relatively great accuracy. The following study shall deal with efforts to classify the teschenite rocks into such a system.

2. Seven groups have been defined in this file which obviously represent basic petrochemical rock types in the file studied. The groups have been defined by basic statistic parameters and variability limits can be determined within the groups and between them. The analytic data file structure can be expressed with sufficient reliability by means of the first two principal components.

3. There are two basic trends of changes in teschenite association rock chemism. One of them - most likely a differentiating trend - causes an increase of total Fe, Ti and P contents with a parallel decrease of Si content. In the direction of this trend the groups 5, 6 and 7 differ from the groups 1 to 4. The second trend - composition of groups shows its assimilation nature - can be seen in an increase of Mg contents with parallel drop of alkali and aluminium contents. In this direction the samples are classified into groups 1 to 4.

4. The results of multi-dimensional chemical composition analysis complement the petrogenetic model of the teschenite association rock genesis. The model presumes the formation of partial melt derived from the upper mantle material and its ascent, or melting through in form of diapirs into the lower part of the Earth's crust with parallel assimilation of melted material. Another ascent occurs during parallel division of the melt into series of magmatic centres. Subsequently, a differentiation process of the material, formed and varied in this way, occurs. This development is expected to have taken place in the initial riftogenesis conditions which is proved, for instance, by linearity of occurrences of these rocks, by trace element distribution, and by a certain analogy of macrochemical development of these rocks with rocks of ocean rifts.

*K tisku doporučil B. Šmíd  
Přeložil A. Farlák*

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# Matematicko-statistická studie chemického složení hornin těšínitové asociace

(Resumé anglického textu)

Pavel Machek - Dalibor Matýsek

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Horniny těšínitové asociace se vzhledem ke svému komplikovanému minerálnímu a chemickému složení, velké strukturní a texturní variabilitě, jakož i značnému stupni jejich ovlivnění sekundárními vlivy velmi obtížně zařazují do klasifikačního systému. Předložená práce je pokračováním pokusů o vytvoření klasifikace těchto hornin na základě jejich chemického složení.

Za základ třídění byla zvolena hierarchická shlukovací analýza s Wardovou-Wishartovou strategií shlukování. Pro matematicko-statistické zpracování bylo vybráno celkem 122 chemických analýz převzatých z literatury. Datový soubor byl standardizován. Výsledky hierarchického shlukování byly dále konfrontovány s výsledky fuzzy shlukování a s analýzou hlavních komponent.

Byla tak vytvořena nová klasifikace hornin těšínitové asociace, třídící tyto horniny do sedmi skupin. Strukturu souboru reprezentuje systém s trojúhelníkovou polaritou. Jedním extrémem jsou horniny těšínitového složení, druhý extrém představují pikrity a poslední extrém pak tvoří fourchity a monchiquity. Výsledky potvrzují, že existují dva základní trendy změn chemismu hornin těšínitové asociace, a to s největší pravděpodobností trend diferenciativní a trend asimilační.

Výsledky vícerozměrné analýzy chemického složení doplňují petrografický model vzniku těšínitové asociace.

## Vysvětlivky k tabulkám

1. Přehled názorů na klasifikaci hornin těšínitové asociace (B. Šmíd, 1978).
2. Vstupní matice - chemické analýzy hornin těšínitové asociace. Teschenite O - olivinický těšínit, C - analcimický těšínit, B - biotitický těšínit, N - nefelinický těšínit, X - pyroxenický těšínit, AX - amfibol-pyroxenický těšínit; picrite T - těšínitický pikrit; monchiquite A - amfibolický monchiquit, B - biotitický monchiquit; fourchite B - biotitický fourchit, AB - amfibol-biotitický fourchit, A - amfibolický fourchit. Zdroje analytických údajů jsou uvedeny v textu.
3. Dendrogram hornin těšínitové asociace (podle Wardovy-Wishartovy strategie shlukování).
4. Rozčlenění souboru do skupin a jejich statistická charakteristika. Skupina 1-7 - průměrná hodnota, odchylka, variační koeficient.
5. Vlastní čísla a vlastní vektory kovariační matice, sestavené pro vstupní údaje standardizované pomocí minimálních a maximálních hodnot.
6. Chemické analýzy alkalických syenitů.

## Vysvětlivky k obrázkům

1. Diagram první a druhé hlavní komponenty. T - těšínit analcimický, nefelinický, O - olivinický těšínit, X - pyroxenický, amfibolický a biotitický těšínit, P - pikrit, F - fourchit, M - monchiquity, 1-7 - čísla shluků.
2. Izolinie obsahu hlavních a některých stopových prvků v grafech první a druhé hlavní komponenty. Vyznačeny jsou skupiny 1 až 7 podle označení na obr. 1.
3. Variační rozpětí a průměrné distribuční křivky obsahu TR pro některé vyčleněné skupiny hornin těšínitové asociace. Analýzy převzaty z práce J. Kuděláskové (1987). Průměrná distribuce TR ve skupině: 1 - pikrity, 2 - pyroxeny, 3 - monchiquity. Normalizace podle J.A. Haskina et al. (1968).
4. Interpretace faktorů variability na základě první a druhé hlavní komponenty. Projekce průměrného složení: 1 - pyroxen, 2 - amfibol, 3 - biotit, 4 - apatit + magnetit.