



### Fractional crystallization – major elements

Mass balance:  $C_{PM} = FC_{EM} + (1-F)C_{cum}$

Lever rule:  $f_{fc} = \frac{C_{PM}^i - C_{EM}^i}{C_{cum}^i - C_{EM}^i}$

$C_{PM}$  = concentration in the parental melt  
 $C_{DM}$  = concentration in differentiated melt  
 $F$  = fraction of the melt remaining (1→0);  
 $f_{fc}$  = (1 - F) = degree of fractional crystallization

(after Cox et al. 1979)

Trends produced by fractional crystallization of cumulate (cum) consisting of one (P), two (P-Q) or three (P-Q-R) minerals. PM = primary melt, FM = fractionated magma

### Fractional crystallization – major elements

- Binary plots of major elements vs. SiO<sub>2</sub> (Harker plots) or any other index of fractionation (e.g., MgO or mg#) often show linear relationships.
- These trends on their own do not provide evidence for operation of fractional crystallization!
- Partial melting or mixing would also produce linear trends (Wall et al. 1987).
- However, the changes in fractionating assemblage result in inflections. If present, they prove operation of fractional crystallization.

(after Wilson 1989)

Harker plots for a suite of cogenetic volcanic rocks developing by fractional crystallization of olivine, clinopyroxene, plagioclase and apatite.

### Fractional crystallization – major elements

Continuously evolving cumulate

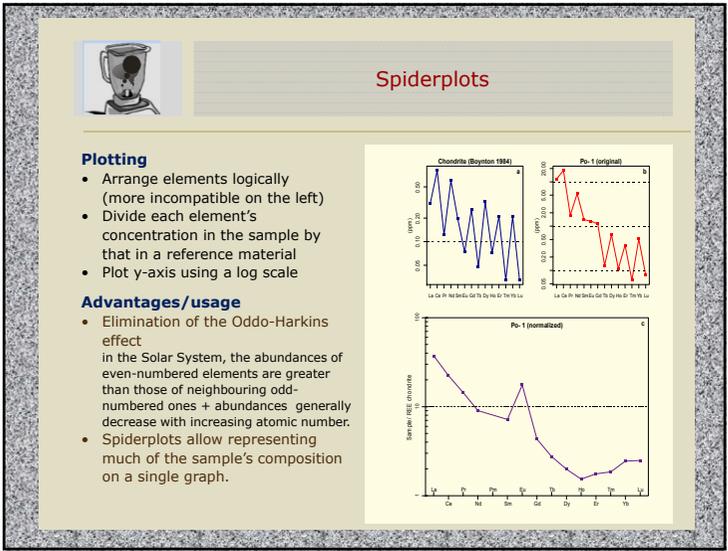
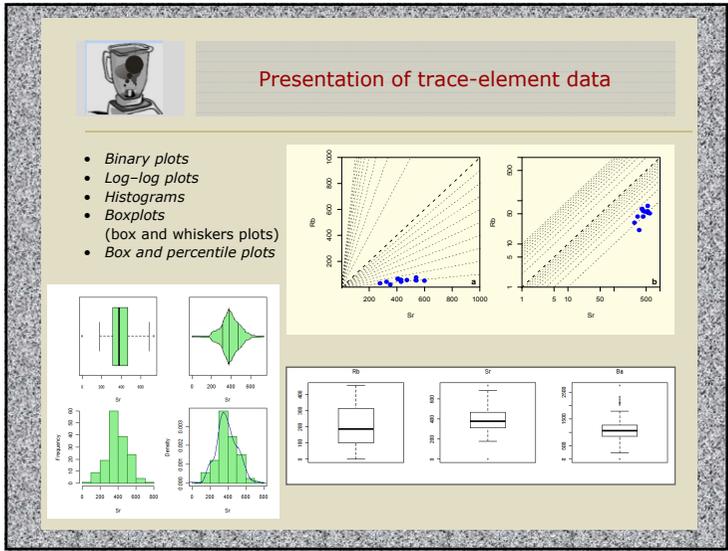
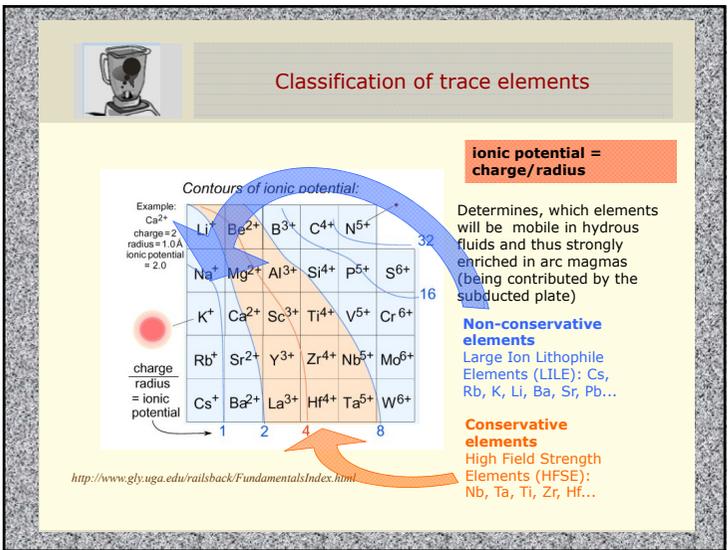
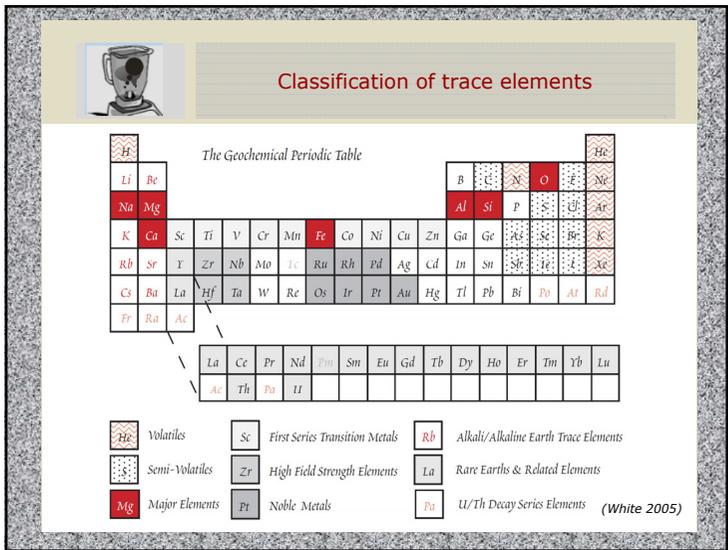
- Produces curved trends.

Reverse modelling

- For reverse modelling of fractional crystallization, the composition of the primitive magma is considered to be a mixture of differentiated magma and cumulus crystals.

$$C_{PM}^i = C_{cum}^i f_{fc} + C_{EM}^i (1 - f_{fc})$$

- Numerical solution is provided, for instance, by the least-squares method (Bryan et al. 1969).

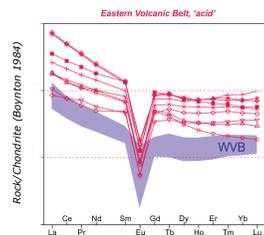
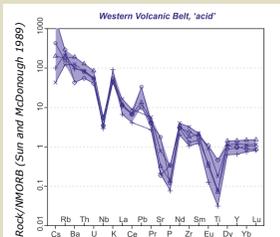




## Spiderplots

### Standards for normalization

- Chondrites ("Bulk Silicate Earth")
- Primitive Mantle
- Mid-Ocean Ridge Basalts (NMORB)
- Ocean-Island Basalts (OIB)
- Averages of various crustal reservoirs, bulk, upper, lower...
- Ocean Ridge Granites (ORG)



Multielement plots for metavolcanic rocks from the Devonian Vrbno Group, Silesia (Czech Republic) (Janoušek et al. 2014)



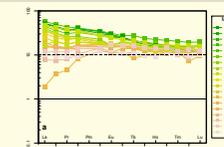
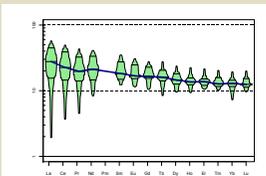
## Spiderplots - examples

### Standards for normalization

- Most primitive/least-altered sample

### Modifications

- Fields
- Colour-coded by a certain parameter (SiO<sub>2</sub>, MgO...)
- Spider box plots
- Spider box and percentile plots



Chondrite-normalized (Boynnton 1984) REE patterns for dolerites from the Devonian Vrbno Group, Silesia



## Two types of trace elements

### "Dilute" trace elements (element partitioning)

- Trace elements that occur in small amounts in the crystal lattice, and their activity is proportional to their concentration (Henry 1803).
- The coefficient of proportionality depends on the nature of the mineral but not on the concentration of the element.
- Modelled by element partitioning between crystals and liquid.

### Essential Structural Constituents (solubility concept)

- *Essential Structural Constituents (ESC)* form a substantial part of the crystal lattice of certain accessory minerals.  
e.g., Zr in zircon, P in apatite or P, Th and LREE in monazite
- The Henry's Law is not applicable.
- Empirical approach, using experimentally determined mineral solubility in a silicate melt.



## Behaviour of accessories - solubility concept

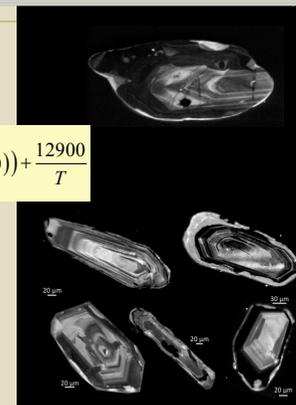
Example - zircon  
(Watson and Harrison 1983)

$$\ln \left( \frac{C_{Zr}^{Zr}}{C_{L}^{Zr}} \right) = (-3.80 - (0.85(M-1))) + \frac{12900}{T}$$

$T$  = temperature in K  
 $C_{Zr}^{Zr}$  = 497644 ppm (49.7 wt. %) is the amount of Zr in an ideal zircon

$M$  = whole-rock chemical parameter based on cation fractions:

$$M = \frac{Na + K + 2Ca}{Al \times Si}$$





## Partition coefficient

### Partition coefficient

$$K_D^{min/L} = \frac{C_{min}}{C_L}$$

**compatible elements**  $K_D \gg 1$   
concentrate into the mineral rather than in the melt

**incompatible elements**  $K_D \ll 1$   
remain in the melt

Does not depend on:

- concentration of the given element
- concentration of any other elements in the system

Measurement:

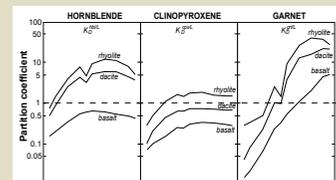
- (Bulk analysis of mineral separates and glass or matrix)
- In-situ measurement in phenocrysts and glass (ion probe, LA ICP MS,...)
- Experiments (doped material)
- Calculations (lattice strain model)

Databases of  $K_D$  values:

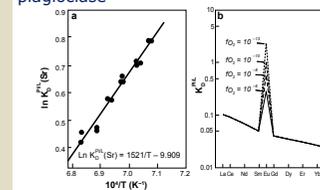
- Rollinson (1993)
- GERM website (<http://earthref.org/KDD/>).



## Partition coefficient



### plagioclase



Depends on:

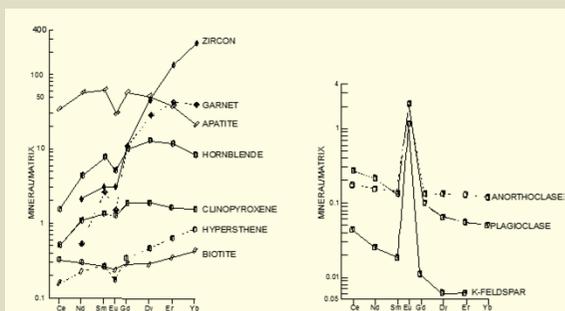
- magma composition (acid/basic, water contents...)
- temperature
- pressure
- mineral
- mineral stoichiometry (composition)
- oxygen fugacity (Eu in plagioclase)

Bulk distribution coefficient

$$D = \sum_i Kd_i X_i$$



## Partition coefficients



Mineral/melt distribution coefficients for REE in dacites and rhyolites (Hanson 1978)

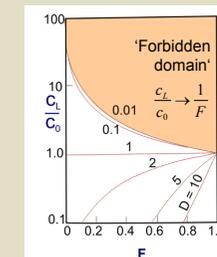


## Fractional crystallization

### Rayleigh equation

$$\frac{C_L}{C_0} = F^{(D-1)}$$

$C_0$  = concentration in the parental melt  
 $C_L$  = concentration in differentiated melt  
 $F$  = fraction of the melt remaining (1-0);  
 $(1-F)$  = degree of fc  
 $D$  = bulk distribution coefficient for the crystallizing phases



Instantaneous solid:

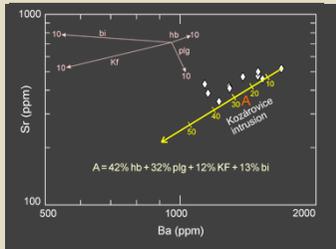
$$c_s^{inst} = Dc_L = Dc_0 F^{(D-1)}$$

Bulk cumulate:

$$\bar{c}_s = c_0 \frac{1 - F^D}{1 - F}$$



## Fractional crystallization



The identification of fractionating phases is facilitated by log-log plots of whole-rock trace-element concentrations, in which the originally exponential Rayleigh trends are converted to linear ones:

$$\log(c_L) = \log(c_0) + (D - 1) \log(F)$$

For granitoids are commonly used Rb, Sr, Ba whose distribution coefficients ( $K_D$ ) are relatively well known (Hanson 1978).

Ba vs Sr patterns for the Kozárovec (diamonds) and Blatná (squares) intrusions of the Central Bohemian Plutonic Complex, Czech Republic (Janoušek et al. 2000)

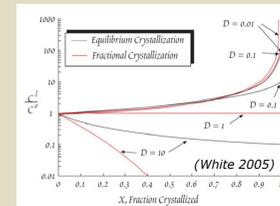


## Equilibrium crystallization

Melt development:

$$C_L = \frac{C_0}{D + F(1 - D)}$$

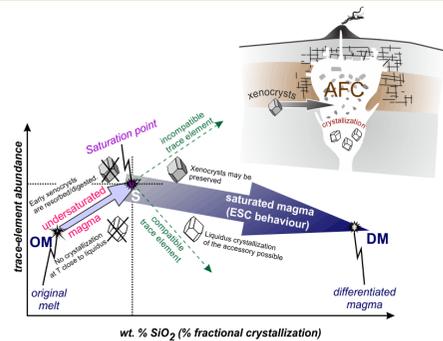
$C_0$  = concentration in the parental melt  
 $C_L$  = concentration in differentiated melt  
 $F$  = fraction of the melt remaining (1→0);  
 $(1 - F)$  = degree of fractional crystallization  
 $D$  = bulk distribution coefficient for the crystallizing phases



Regardless of its exact mechanism (fractional/equilibrium), crystallization quickly depletes compatible elements from the melt.



## Fractional crystallization – behaviour of accessories



Watson & Harrison (1984); Hoskin et al. (2000); Janoušek (2006)



## Fractional melting

Melt development:

$$\frac{C_S}{C_0} = (1 - F)^{\left(\frac{1}{D} - 1\right)}$$

$C_0$  = concentration in the (unmelted) source  
 $C_L$  = concentration in the melt  
 $F$  = degree of melting  
 $D$  = bulk distribution coefficient **after melting** (residue)

At any moment, the instantaneous liquid equilibrates with the solid residue. The composition of a single melt increment is:

$$\frac{C_L}{C_0} = \frac{1}{D} (1 - F)^{\left(\frac{1}{D} - 1\right)}$$

Average composition of the (aggregated) melt:

$$\frac{C_L}{C_0} = \frac{1}{F} \left( 1 - (1 - F)^{\frac{1}{D}} \right)$$



### Batch melting

Melt development:

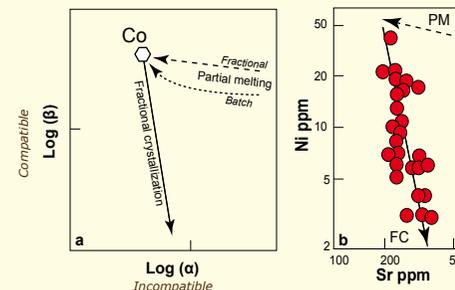
$$C_L = \frac{C_0}{D + F(1-D)}$$

$C_0$  = concentration in the (unmelted) source  
 $C_L$  = concentration in the melt  
 $F$  = degree of melting  
 $D$  = bulk distribution coefficient **after melting** (residue)

All the melt is formed, and then separated in a single batch.



### Distinguishing between fractional crystallization and partial melting



Fractional crystallization (FC) produces an almost vertical trend whereas partial melting (PM) results in a nearly horizontal one (after Martin 1987).



### (Binary) mixing

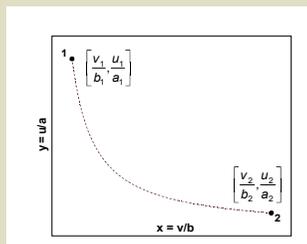
Mass-balance equation:

$$C_M = \sum_{k=1}^m (f_k C_k)$$

$C_M$  = concentration in the mixture  
 $C_k$  = concentration in the end-member  $k$   
 $f_k$  = proportion of the given end-member  $k$

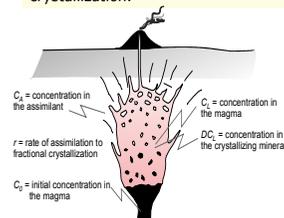
Trends in binary plots:

- **Element-element:** straight line
- **Element-ratio:** hyperbola
- **Ratio-ratio:** hyperbola [general]
- **Ratio-ratio:** straight line [common denominator]



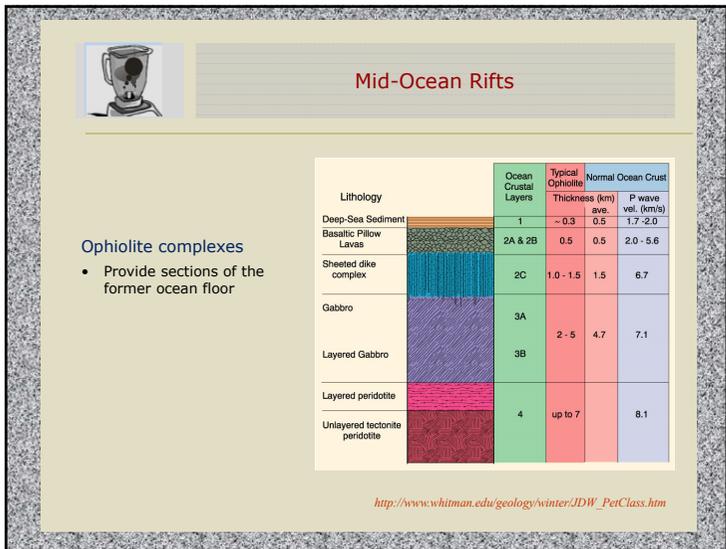
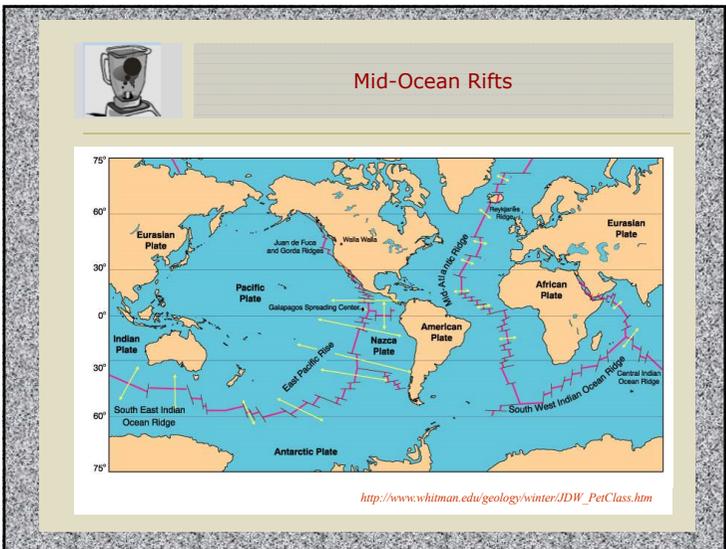
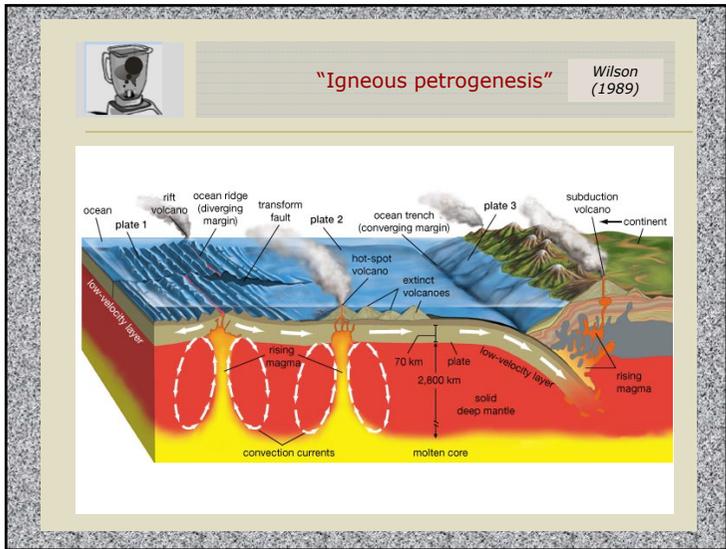
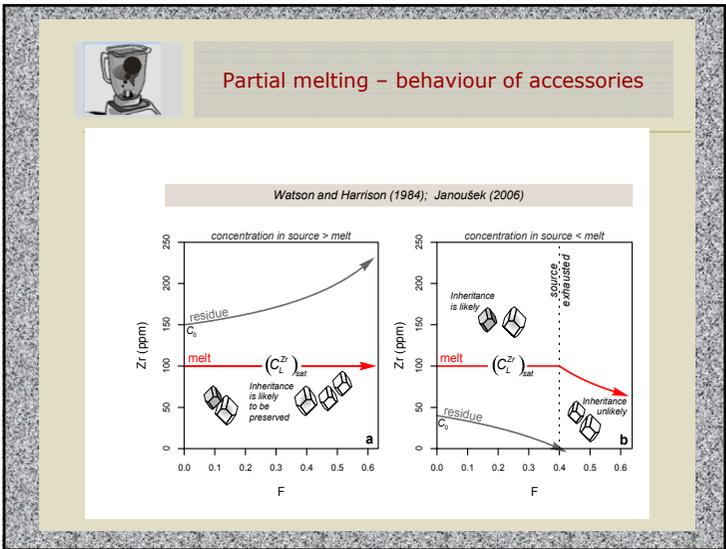
### Combined Assimilation and fractional crystallization (AFC)

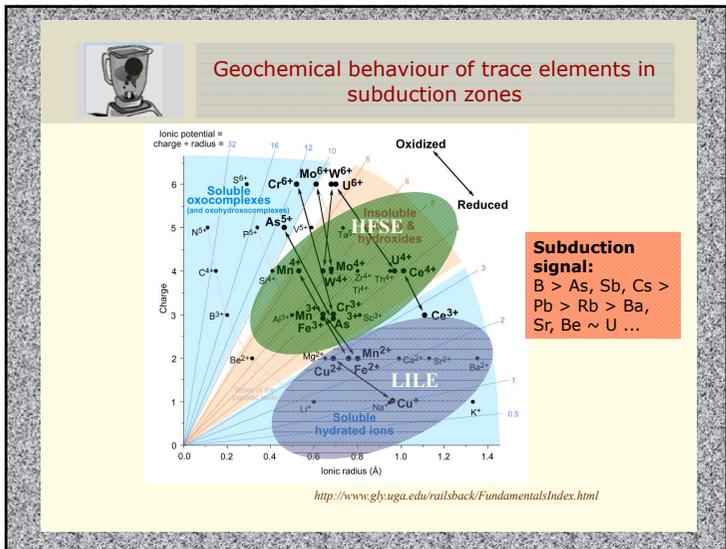
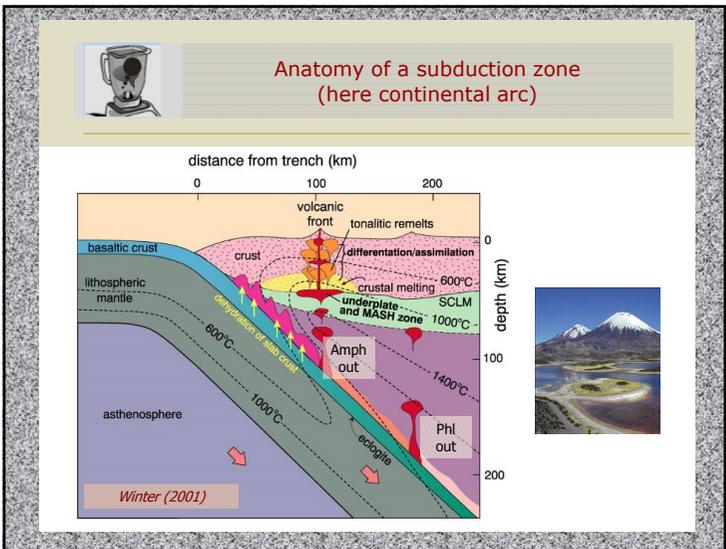
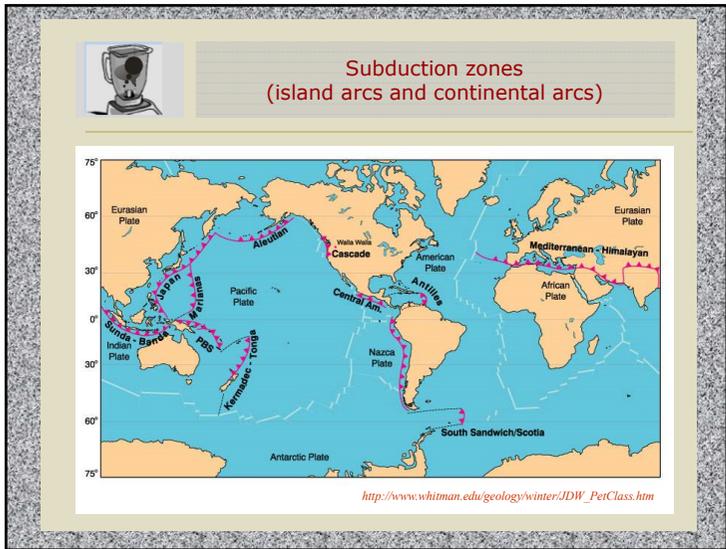
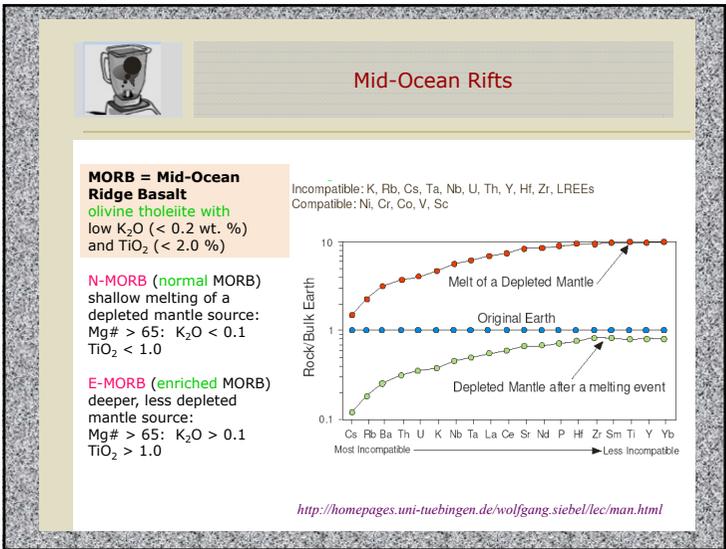
The AFC model assumes that the extra heat needed for assimilation (which is an endothermic process) is provided by the latent heat of crystallization.



Direct AFC models can be calculated and plotted by Petrograph (Petrelli et al. 2005) or special spreadsheets (Ersoy and Helvacı 2010; Keskin 2013).

- The sophisticated AFC equations with many parameters can be easily tweaked to yield solutions nicely reproducing the observed variation but geologically unrealistic (Roberts and Clemens 1995)







### Continental vs. island arcs

#### Main differences of continental from island arcs:

- Greater scope for **crustal contamination** while mantle-derived magmas ascend through the thick, geochemically and isotopically evolved continental crust.
- Low density of the crust acts as a **"density filter"**, i.e. it decreases the ascent velocity (or even stops) the rising basic melts
- Their **stagnation leads to fractional crystallization and assimilation (MASH = Melting Assimilation Storage Homogenization; Hildreth and Moorbath 1988)** at the mantle-lower crust boundary (at about the Moho depth)

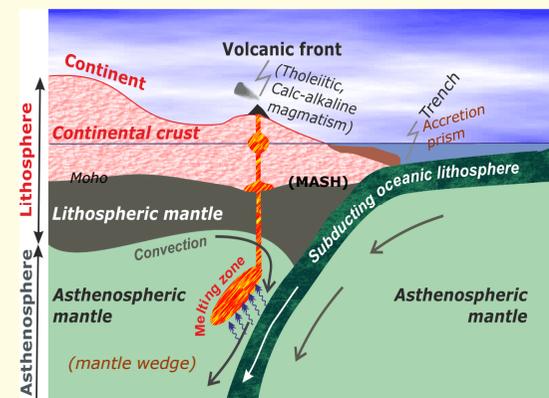


Cotopaxi

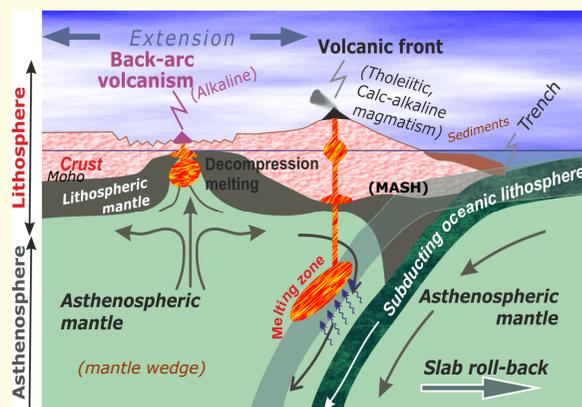
- Low solidus temperature facilitates **partial melting of the fertile continental crust**



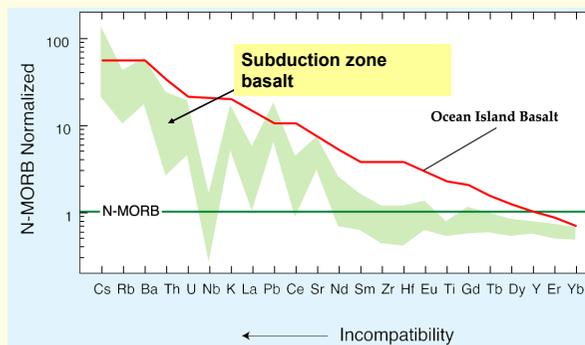
### Anatomy of a subduction zone (compression)



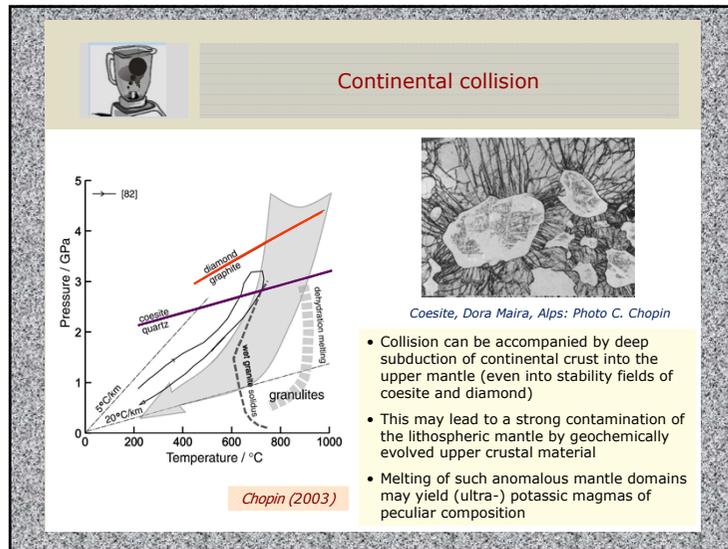
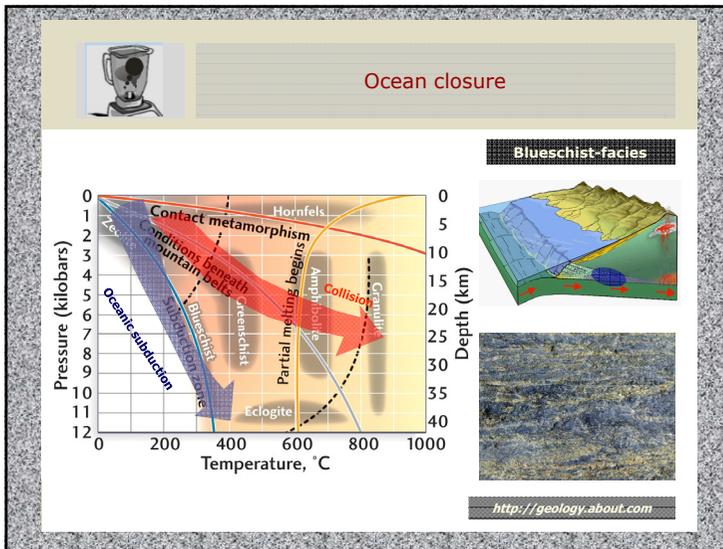
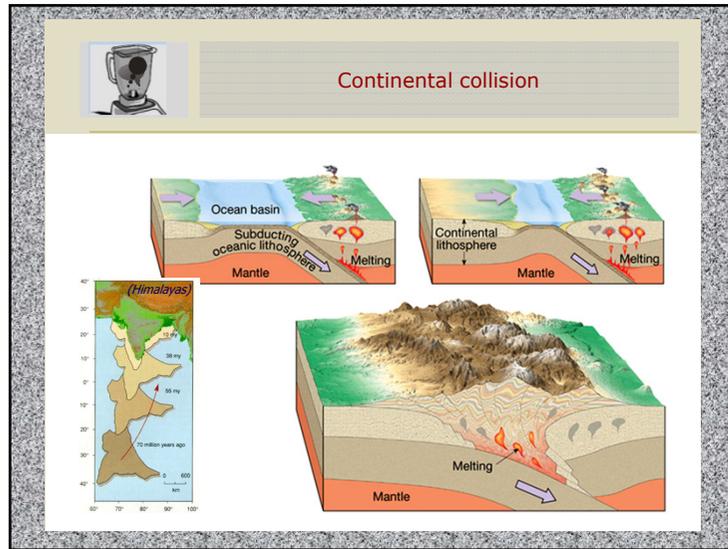
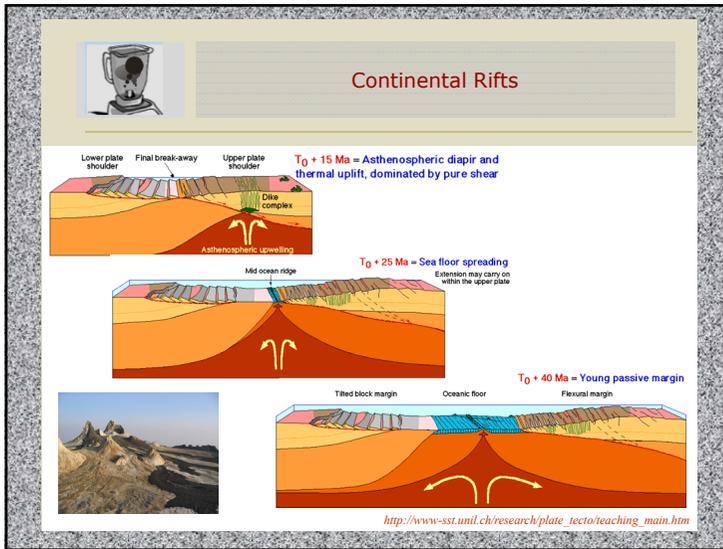
### Anatomy of a subduction zone (extension)



### Trace-element signature of subduction-related magmas





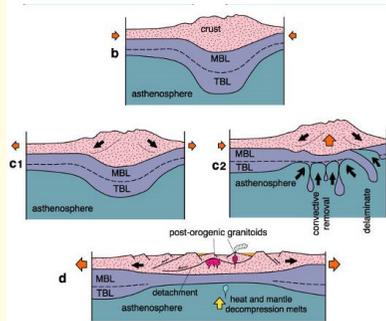




## Post-Collisional Development

### Heat for crustal anatexis

- **Stacking/thickening**  
*in-situ* heat production by radioactive decay of K, Th, U
- **Advection of heat** by quickly exhumed hot lower crustal rocks or intruding basic magmas
- **Conduction of heat from a thermal anomaly in the mantle** (slab break-off, mantle delamination, asthenosphere upwelling, a mantle plume...),
- **Conduction of heat from anomalous mantle** – *in situ* radioactive decay of K, Th, U in crustally contaminated lithospheric mantle.



[http://www.whitman.edu/geology/winter/JDW\\_PetClass.htm](http://www.whitman.edu/geology/winter/JDW_PetClass.htm)



## Models for petrogenesis of granitoid rocks

- **Partial melting of crustal rocks**
  - Regional metamorphism – deep burial (granulite-facies)
  - Injection of basic magma, basic magma underplating
  - Crustal thickening or thinning
  - Decompression melting during uplift of crustal rock complexes
- **Contamination of mantle-derived magmas**  
assimilation of crustal material, followed by differentiation
- **Differentiation of mantle-derived magmas**  
mainly by fractional crystallization
- **Dehydration and partial melting of hydrated oceanic crust (including sediments) in subduction zones**  
fluids and small-scale melts move upwards and trigger melting of the overlying mantle wedge
- **Partial melting of (meta-) basic rocks (amphibolites etc.)**  
previous magma pulses, relicts of the subducted oceanic crust...



## Petrogenetic classification of granitoid rocks

### Alumina saturation and mineralogy

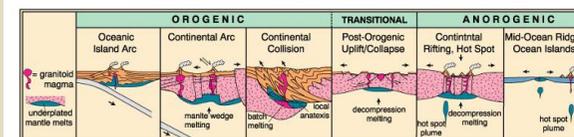
Clarke (1992)

	Peraluminous	Metaluminous	Peralkaline
<b>Definition</b>	A > CNK	CNK > A > NK	A < NK
<b>Characteristic minerals</b>	alumosilicates, cordierite, garnet, topaz, tourmaline, spinel, corundum	Pyroxene, amphibole, epidote	Fe-rich olivine (fayalite), aegirine, arfvedsonite, riebeckite
<b>Other common minerals</b>	biotite, muscovite	biotite, muscovite rare	rare biotite
<b>Fe-Ti oxide phase</b>	Ilmenite	Magnetite	Magnetite
<b>Accessories</b>	apatite, zircon, monazite	apatite, zircon, titanite, allanite	apatite, zircon, titanite, allanite, fluorite, cryolite, pyrochlore
$(^{87}\text{Sr}/^{86}\text{Sr})_i$	0.705–0.720	0.703–0.708	0.703–0.712
$\epsilon_{\text{Nd}}$	<< 0	~ 0	variable



## Petrogenesis of granitic rocks

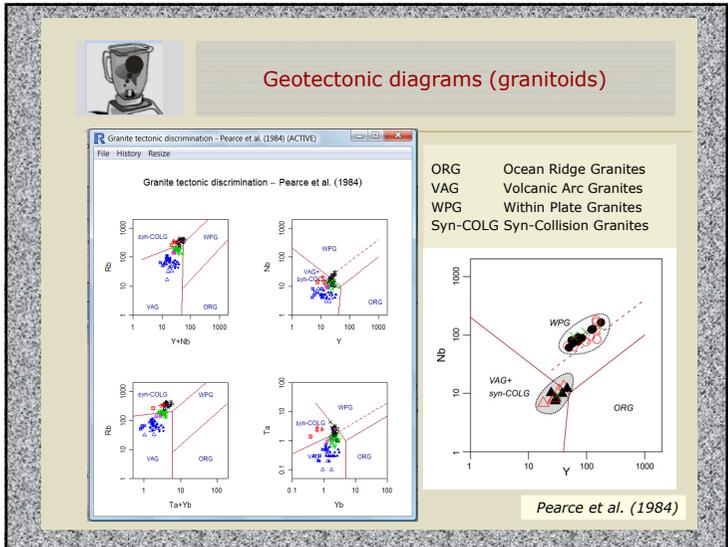
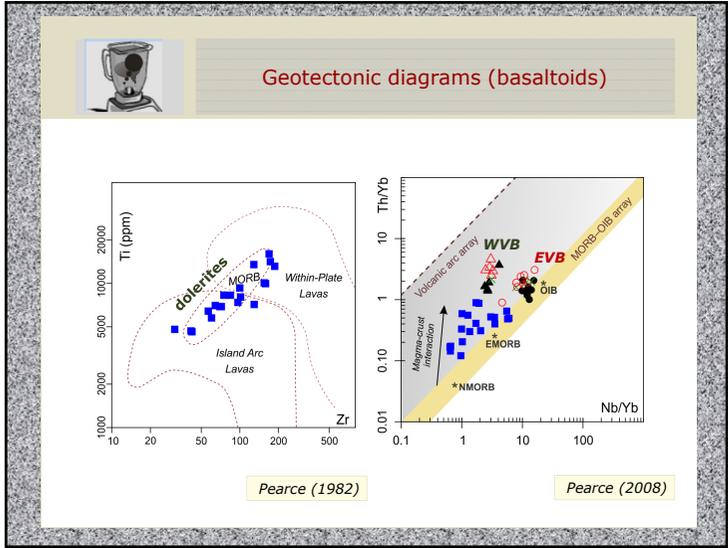
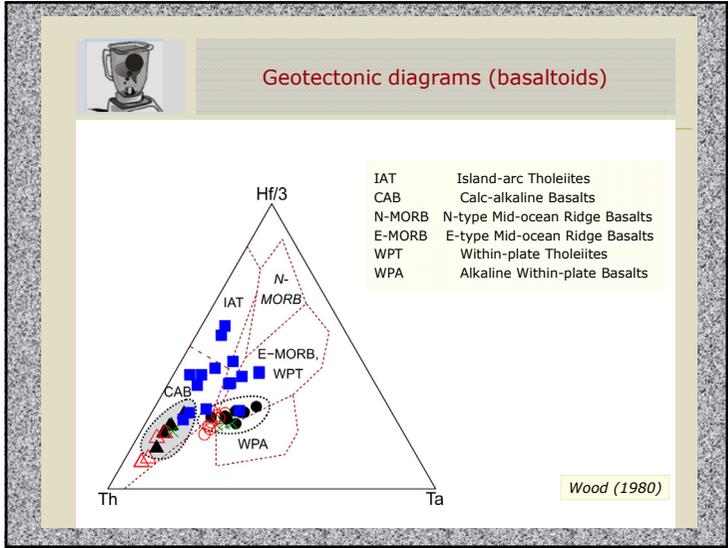
Table 18-4. A classification of granitoid rocks based on tectonic setting



Pitcher (1983), Barbarin (1990), Pitcher (1993)

[http://www.whitman.edu/geology/winter/JDW\\_PetClass.htm](http://www.whitman.edu/geology/winter/JDW_PetClass.htm)





### References and further reading

ALBARÈDE F. 1995. Introduction to the Geochemical Modeling. Cambridge University Press.

BARBARIN, B., 1990. Granitoids: main petrogenetic classifications in relation to origin and tectonic setting. *Geological Journal*, **25**, 227-238.

BOYNTON, W. V., 1984. Cosmochemistry of the rare earth elements: meteorite studies. In: Henderson, P. (ed.): Rare Earth Element Geochemistry. Elsevier, Amsterdam, 63-114.

BRYAN W.B., FINGER L.W. & CHAYES F. 1969. Estimating proportions in petrographic mixing equations by least-squares approximation. *Science* **163**: 926-927.

CHOPIN, C., 2003. Ultrahigh-pressure metamorphism: tracing continental crust into the mantle. *Earth and Planetary Science Letters*, **212**, 1-14.

CLARKE, D.B. 1992. Granitoid Rocks. Chapman & Hall, London.

COX K.G., BELL J.D. & PANKHURST R.J. 1979. The Interpretation of Igneous Rocks. George Allen & Unwin, London.

DEPAOLO, D.J. 1981. Trace element and isotopic effects of combined wallrock assimilation and fractional crystallization. *Earth Planet. Sci. Lett.*, **53**, 189-202.

ERSOY, Y. & HELVACI, C., 2010. FC-AFC-FCA and mixing modeler: a Microsoft® Excel® spreadsheet program for modeling geochemical differentiation of magma by crystal fractionation, crustal assimilation and mixing. *Computers & Geosciences*, **36**, 383-390.

HANSON, G. N., 1978. The application of trace elements to the petrogenesis of igneous rocks of granitic composition. *Earth and Planetary Science Letters*, **38**, 26-43.

HANSON G.N. 1980. Rare earth elements in petrogenetic studies of igneous systems. *Ann. Rev. Earth Planet. Sci.* **8**: 371-406.



## References and further reading

- HILDRETH, W. & MOORBATH, S., 1988. Crustal contributions to arc magmatism in the Andes of Central Chile. *Contributions to Mineralogy and Petrology*, **98**, 455-489.
- JANOUSEK, V., 2006. Saturnin, R language script for application of accessory-mineral saturation models in igneous geochemistry. *Geologica Carpathica*, **57**, 131-142.
- JANOUSEK, V. *et al.* 2000. Modelling diverse processes in the petrogenesis of a composite batholith: the Central Bohemian Pluton, Central European Hercynides. *Journal of Petrology*, **41**, 511-543.
- JANOUSEK, V. *et al.* 2014. Constraining genesis and geotectonic setting of metavolcanic complexes: a multidisciplinary study of the Devonian Vrbno Group (Hrubý Jeseník Mts., Czech Republic). *International Journal of Earth Sciences*, **103**, 455-483.
- KESKIN, M., 2013. AFC-Modeler: a Microsoft® Excel® workbook program for modelling assimilation combined with fractional crystallization (AFC) process in magmatic systems by using equations of DePaolo (1981). *Turkish Journal of Earth Sciences*, **22**, 304-319.
- MARTIN, H., 1987. Petrogenesis of Archaean trondhjemites, tonalites, and granodiorites from eastern Finland: major and trace element geochemistry. *Journal of Petrology*, **28**, 921-953.
- MCDONOUGH, W. F. & SUN, S., 1995. The composition of the Earth. *Chem. Geology*, **120**, 223-253.
- PEARCE, J. A., 1982. Trace element characteristics of lavas from destructive plate boundaries. In: THORPE, R. S. (ed.): *Andesites; Orogenic Andesites and Related Rocks*. John Wiley & Sons, Chichester, 525-548.
- PEARCE, J. A., 2008. Geochemical fingerprinting of oceanic basalts with applications to ophiolite classification and the search for Archean oceanic crust. *Lithos*, **100**, 14-48.



## References and further reading

- PEARCE, J. A., HARRIS, N. B. W. & TINDLE, A. G., 1984. Trace element discrimination diagrams for the tectonic interpretation of granitic rocks. *Journal of Petrology*, **25**, 956-983.
- PETRELLI, M. *et al.* 2005. PetroGraph: a new software to visualize, model, and present geochemical data in igneous petrology. *Geochemistry, Geophysics, Geosystems*, **6**, Q07011.
- PITCHER, W. S., 1983. Granite type and tectonic environment. In: HSÜ, K. J. (ed.): *Mountain Building Processes*. Academic Press, London, 19-40.
- PITCHER, W.S. 1993. *The Nature and Origin of Granite*. Chapman & Hall, London.
- ROBERTS, M. P. & CLEMENS, J. D., 1995. Feasibility of AFC models for the petrogenesis of calc-alkaline magma series. *Contributions to Mineralogy and Petrology*, **121**, 139-147.
- ROLLINSON H.R. 1993. *Using Geochemical Data: Evaluation, Presentation, Interpretation*. Longman, London.
- WALL, V.J., CLEMENS, J.D. & CLARKE, D.B. 1987. Models for granitoid evolution and source compositions. *The Journal of Geology*, **95**, 731-749.
- WHALEN J.B., CURRIE K.L., CHAPPELL B.W. (1987) A-type granites: geochemical characteristics, discrimination and petrogenesis. *Cont. Mineral. Petrol.*, **95**, 407-419.
- WATSON, E. B. & HARRISON, T. M., 1983. Zircon saturation revisited: temperature and composition effects in a variety of crustal magma types. *Earth and Planetary Science Letters*, **64**, 295-304.
- WATSON, E. B. & HARRISON, T. M., 1984. Accessory minerals and the geochemical evolution of crustal magmatic systems: a summary and prospectus of experimental approaches. *Physics of the Earth and Planetary Interiors*, **35**, 19-30.



## References and further reading

- WILSON, M., 1989. *Igneous Petrogenesis*. Unwin Hyman, London.
- WINTER, J. D., 2001. *An Introduction to Igneous and Metamorphic Geology*. Prentice Hall, Upper Saddle River, NJ.
- WOOD, D. A., 1980. The application of a Th-Hf-Ta diagram to problems of tectonomagmatic classification and to establishing the nature of crustal contamination of basaltic lavas of the British Tertiary volcanic province. *Earth and Planetary Science Letters*, **50**, 11-30.

## Web links

- Geochemistry 455 (W.M. White, Cornell University)  
<http://www.geo.cornell.edu/geology/classes/geo455/Geo455.html>
- Igneous and metamorphic geology (J. D. Winter, Whitman University)  
[http://www.whitman.edu/geology/winter/JDW\\_PetClass.htm](http://www.whitman.edu/geology/winter/JDW_PetClass.htm)
- Advanced petrology (J.-F. Moyen, University of Stellenbosch, South Africa)  
[http://academic.sun.ac.za/earthSci/honours/modules/igneous\\_petrology.htm](http://academic.sun.ac.za/earthSci/honours/modules/igneous_petrology.htm)
- EarthRef.org. The website for Earth Science reference data and models  
<http://earthref.org/>



## References and further reading

- Janoušek V., Moyen J.-F., Martin H., Erban V., Farrow C. (in print): *Geochemical Modelling of Igneous Processes – Principles And Recipes in R Language*. Springer Geochemistry 1. Springer, Berlin (July 2015)

